

THE EFFECT OF INTERNAL FLOW ON MARINE RISER DYNAMICS

By

NAMSEEG HONG

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1994

ACKNOWLEDGMENTS

I stood at a turning point of life two years ago. When I look back the moment, it was a very difficult time for me and my family to endure even though I feel comfortable now. I might have given up my degree and returned to my country without their help. Their encouragement and direction have been essential factors in my achievement and a motive of this work. I would like to express my most grateful thanks to Dr. Robert G. Dean who was my chairman and to Dr. Hsiang Wang who gave me guidance for the completion of this dissertation.

My appreciation is also extended to my committee members, Dr. Robert J. Thieke, Dr. D. Max Sheppard, and Dr. C.O. Hays. I am also grateful to Dr. Ashish J. Metha, Dr. Y. Peter Sheng, Michael K. Ochi, and Dr Daniel M. Hanes. I owe a great debt to Dr. Chul-Hee Yoo, Dr. Jung Lyul Lee, and Dr. Jei-Kook Choi, who shared their knowledge with me from the beginning. Specially, more thanks go to Dr. Yoo for his timely advice. Also, thanks go to Mr. Surbarna Malakar for assisting me in installing my PC, and to Mrs Lucy Stevens in Family Housing. I am also indebted to Mrs. Becky Hudson, Mrs. Sandra Bivins, Mrs. Lucy Hamm, Mrs. Helen Twedell, Mrs. Cynthia Vey, and Mr. John Davis.

Many thanks, and my fortune, go to my wife Mrs. Geumjoo Hong and my lovely daughter Minsu Hong for their smiles, patience, sacrifice, love and for not asking to go back to my country in the middle of my study. I must thank my parents, Jinil Hong (Soon Go Hong) and Myeo-Hee Lee for their unlimited support throughout my life, financially and spiritually. In particular, I will never forget my father's sacrifice for his children.

TABLE OF CONTENTS

ACKNOWLEDGMENTS	ii
LIST OF FIGURES	vi
LIST OF TABLES	xii
ABSTRACT	xiii
CHAPTERS	
1 INTRODUCTION	1
1.1 General Description	1
1.2 Literature Review	4
1.2.1 Instability of Fluid-Conveying Pipe	4
1.2.2 The Effect of Internal Flow on Riser Dynamics	6
1.2.3 Riser Analysis	7
1.3 Scope of Study	9
2 MATHEMATICAL MODEL OF THE SYSTEM	13
2.1 Simplifications and Assumptions of Model	14
2.2 Equations of the Pipe	16
2.3 Force due to Internal Flow	22
2.4 Hydrodynamic Forces	25
2.5 Equations of Motion and Boundary Conditions	28
2.5.1 Nonlinear Governing Equations	28
2.5.2 Boundary Conditions	30
2.5.3 Approximate Governing Equations and B.C.s	32
2.5.4 Current-Vortex Model	34

3	FINITE ELEMENT MODEL OF THE SYSTEM	37
3.1	Variational Statement for the Problem	37
3.2	Incremental Weak Formulation	40
3.3	Finite Element Approximations	44
3.4	Matrix Dynamic Equilibrium Equation	51
3.5	Finite Element Model of Linear System	59
4	DEVELOPMENT OF A FINITE ELEMENT PROGRAM	62
4.1	Description of CODE	62
4.2	Element Calculations	63
4.3	Preprocessing Routines	68
4.4	Finite Element Calculation Routines	71
5	SOLUTION OF FREE VIBRATION OF THE SYSTEM	73
5.1	Semi-Analytical Solution	73
5.1.1	Quasi Mode Shape	74
5.1.2	Modified Eigenvalue Problem	76
5.2	MDOF System with Coriolis Matrix	79
5.3	Numerical Evaluation of System Frequency	81
5.3.1	Algorithm for Computer Program	81
5.3.2	Comparison of Results	82
6	FORCED VIBRATION OF THE SYSTEM	86
6.1	Newmark Method for Time Step Integration	86
6.2	Predictor-Corrector Scheme	89
6.3	Vortex-Induced Vibration Due to Current	92
6.4	Vibration Due to Wave or Current Loading	104
6.5	Modification of the System Matrices or Vector for Top Boundary Condition	116
7	ANALYSIS OF THE EFFECT OF INTERNAL FLOW ON RISER DYNAMICS	121
7.1	The Effect on System Frequency	121
7.2	Comparison between the Effects of Centrifugal and Coriolis Forces	123
7.3	The Effect on Vortex-Induced Vibration	127
7.4	Consideration of Geometric Nonlinearity	134
7.5	The Effect on Vibration Due to Wave or Current Loading	138
7.6	The Effect on Internal Stresses and Displacement Fields	150

8 CONCLUSIONS AND FURTHER STUDY	160
8.1 Conclusions	160
8.2 Further Study	161
APPENDICES	
A DERIVATION OF LINEAR GOVERNING EQUATION USING HAMILTON'S PRINCIPLE	163
B DERIVATION OF FINITE ELEMENT MODEL OF LINEAR SYSTEM	165
BIBLIOGRAPHY	168
BIOGRAPHICAL SKETCH	172

LIST OF FIGURES

1.1	Configuration of Riser System with the Inclusion of Internal Flow.....	3
2.1	Simplified Configuration of the Riser System	14
2.2	Position Vector and Principal Direction at a Point on the Riser Centerline	17
2.3	Velocity Vectors Acting on a Segment of Pipe and Definition of the Relative Velocity Components	25
2.4	Free Body Diagram of the Equilibrium of Forces and Moments for the Differential Element at the Top of the Riser	31
2.5	Control Volume Containing a Cylinder Shedding Vortices	35
3.1	(a) Hermite Cubic Shape Functions on a Master Element and (b) Corresponding Global Basis Functions at a Node i in the Mesh	46
3.2	An Element of Riser Showing Forces and Displacements at the Node	56
4.1	Functional Units of the Finite Element Code	62
4.2	Flow Charts of Components of CODE	64
4.3	A Brief Description of Subroutines	65
4.4	Data Arrays in CODE	66
5.1	Algorithm for Solving Non-Standard Eigenvalue Problem	81
5.2	The Design Properties and Data for a Drilling Riser System for Use in the Northern North Sea	83
6.1	Newmark's Constant Average Acceleration Scheme	87
6.2	Algorithm for Step-by-Step Solution Using Newmark Integration Method	88

6.3	Solution Algorithm Employing Predictor-Corrector Scheme	91
6.4	Flow Charts of Components of Subroutine NEWMRK in CODEV	93
6.5	Maximum Vortex-Induced Displacement in Transverse Direction According to the Variation of Uniform Inline Current. The System Includes Internal Tension and Internal Flow. (Internal Velocity = 6.0 m/sec)	96
6.6	Maximum Inline Displacement in Current Direction According to the Variation of Uniform Inline Current. The System Includes Internal Tension and Internal Flow. (Internal Flow Velocity = 6.0 m/sec)	98
6.7	Transverse Displacement Due to Inline Current at the Middle Node of Riser According to Change of Time. The System Includes Internal Tension and Internal Flow of 6.0 m/sec	99
6.8	Inline Displacement Due to Inline Current at the Middle Node of Riser According to Change of Time. The System Includes Internal Tension and Internal Flow of 6.0 m/sec	100
6.9	Time-Dependent Trajectories of Displacement Vector Tip at the Middle Node of Riser. The System Includes Internal Tension and Internal Flow of 6.0 m/sec	101
6.10	Deflected Shape of Riser System in Transverse and Current Direction According to the Change of System. Four Different Combinations Are Considered As Described in Figure. (Current Velocity = 0.40 m/sec, Internal Flow Velocity = 6.0 m/sec)	103
6.11	Flow Chart of Components of Subroutine NEWMRK in CODE	105
6.12	Time Simulations of the Displacement at the Middle of Riser in Current Direction. Risers with Different Length Are Subject to a Uniform Current ($U = 0.4$ m/sec) and a Linear Wave ($H_w = 6.1$ m and $T_w = 7.8$ sec) with a Right Angle at the Same Time. (Dashed Line = Nonlinear Model, Solid Line = Linear Model)	107
6.13	Time Simulations of the Displacement at the Middle of Riser in Wave Direction. Risers with Different Length Are Subject to a Uniform Current ($U = 0.4$ m/sec) and a Linear Wave ($H_w = 6.1$ m and $T_w = 7.8$ sec) with a Right Angle at the Same Time. (Dashed Line = Nonlinear Model, Solid Line = Linear Model)	108
6.14	Displacement Envelope in Current Direction under Current ($U = 0.4$ m/sec) and Wave ($H_w = 6.1$ m and $T_w = 7.8$ sec) with a Right Angle. (Dashed Line = Nonlinear Model, Solid Line = Linear Model)	109
6.15	Displacement Envelope in Wave Direction under Current ($U = 0.4$ m/sec) and Wave ($H_w = 6.1$ m and $T_w = 7.8$ sec) with a Right Angle. (Dashed Line = Nonlinear Model, Solid Line = Linear Model)	110

6.16	Maximum Displacement in Wave Direction According to the Change of Wave Frequency. ($H_w = 6.1$ m)	111
6.17	Time Simulations of the Displacement at the Middle of a 152 m Riser in Wave Direction. ($H_w = 6.1$ m)	112
6.18	Time Trajectories of the Displacement at the Middle of a 152 m Riser under Wave ($H_w = 6.1$ m) and Current ($U = 0.4$ m/sec) with a Right Angle	114
6.19	Time Simulations in Current and Wave Direction and Time Trajectory at the Middle of a 152 m Riser under Current ($U = 1.5$ m/sec) and a Linear Wave ($H_w = 6.1$ m and $T_w = 4$ sec) with a Right Angle. Rayleigh Damping $[C] = 0.01 [M] + 0.03 [K]$ and Top Tension Ratio $TR = 1.2$	115
6.20	Time Simulations in Current and Wave Direction and Time Trajectory at the Top of a 152 m Riser under Current ($U = 1.5$ m/sec) and a Linear Wave ($H_w = 10$ m and $T_w = 16$ sec) with a Right Angle. Rayleigh Damping $[C] = 0.01 [M] + 0.03 [K]$ and Top Tension Ratio $TR = 1.2$	117
6.21	The Displacement Envelopes in Current Direction at a Specific Time for a 152 m Riser under Current ($U = 1.5$ m/sec) and a Linear Wave ($H_w = 10$ m and $T_w = 16$ sec) with a Right Angle. Rayleigh Damping $[C] = 0.01 [M] + 0.03 [K]$ and Top Tension Ratio $TR = 1.2$	118
6.22	The Displacement Envelopes in Wave Direction at a Specific Time for a 152 m Riser under Current ($U = 1.5$ m/sec) and a Linear Wave ($H_w = 10$ m and $T_w = 16$ sec) with a Right Angle. Rayleigh Damping $[C] = 0.01 [M] + 0.03 [K]$ and Top Tension Ratio $TR = 1.2$	120
7.1	Internal Flowing Fluid Effect on 1st Mode System Frequency of a Neutrally Buoyant Riser System. The internal flow velocity changes from 1.5 m/sec to 12.0 m/sec and mean tension (same as top tension) does from null to 50 kN	122
7.2	Comparison Between the Effects of Each Forcing Terms (Centrifugal and Coriolis Forces) Due to Internal Flow on 1st System Frequency. Mean Tension = 50 kN, Internal Flow Velocity = 12 m/sec	124
7.3	The Effect of Centrifugal Force on Displacement Envelope of Riser at an Arbitrary Time. ($TR = 1.2$, $w = 3.86$ kN/m, $H_w = 6.1$ m, $T_w = 15$ sec)	125
7.4	The Effect of Centrifugal Force on Displacement Envelope of Riser at an Arbitrary Time. ($TR = 1.2$, $w = 3.86$ kN/m, $H_w = 6.1$ m, $T_w = 5$ sec)	126
7.5	The Effect of Coriolis Force on Displacement Envelope of Riser at an Arbitrary Time. ($TR = 1.2$, $w = 3.86$ kN/m, $H_w = 6.1$ m, $T_w = 15$ sec)	128

7.6	The Effect of Coriolis Force on Displacement Envelope of Riser at an Arbitrary Time. (TR = 1.2, w = 3.86 kN/m, Hw = 6.1 m, Tw = 5 sec)	129
7.7	The Effect of Internal Flow on the Displacement at the Middle of 152 m Riser. (Current Velocity U = 0.32 m/sec, Top Tension = 1550 kN, The Other Properties are the Same as Example of Fig. 7.6)	130
7.8	The Effect of Internal Flow on the Trajectory at the Middle of 152 m Riser According to the Lapse of Time. (Current Velocity U = 0.32 m/sec, Top Tension = 1550 kN, The Other Properties are the Same as Example of Fig. 7.6)	131
7.9	The Effect of Internal Flow on Maximum Displacement in Vortex Direction for 152 m Riser. Riser Properties except Top Tension are the Same as Example Figure 7.6	133
7.10	The Effect of Internal Flow on Maximum Displacement Envelope in Vortex Direction for 152 m Riser at Some Current Velocity. Top Tension = 1550 kN, Bottom Tension = 1200 kN. Riser Properties except Tension are the Same as Example Figure 7.6. Top Condition is Semi-Restrained	135
7.11	The Effect of Internal Flow on Maximum Displacement Envelope in Vortex Direction for 152 m Riser at Some Current Velocity. Top Tension = 350 kN, Bottom Tension = 0 kN. Riser Properties except Tension are the Same as Example Figure 7.6. Top Condition is Semi-Restrained.	136
7.12	Comparison Between Linear and Nonlinear Model about the Effect of Internal Flow According to the Change of Water Depth and Internal Flow Velocity. Top Tension = 650 kN, TR = 1.1, Effective Weight = Top Tension / (TR x Riser Length)	137
7.13	Comparison Between Linear and Nonlinear Model about the Effect of Internal Flow According to the Change of Water Depth and Internal Flow Velocity. Top Tension = 765 kN, TR = 1.1, Effective Weight = Top Tension / (TR x Riser Length)	139
7.14	The Effect of Internal Flow on the Displacement in Current Direction at the Tip of 152 m Riser. Second Figure Shows the Zoom Picture at Certain Time Interval. (Current Velocity U = 0.40 m/sec, Top Tension = 350 kN, Effective Weight w = 2.3 kN/m, Semi-Restrained Top Condition)	140
7.15	The Effect of Internal Flow on the Displacement in Wave Direction at the Tip of 152 m Riser. Second Figure Shows the Zoom Picture at Certain Time Interval. (Wave Period Tw = 20 sec, Wave Height Hw = 6.1 m, Top Tension = 350 kN, Effective Weight w = 2.3 kN/m, Semi-Restrained Top Condition)	141

7.16	The Effect of Internal Flow on the Time-Dependent Trajectory at the Tip of 152 m Riser. (Current Velocity $U = 0.4$ m/sec, Wave Period $T_w = 20$ sec, Wave Height $H_w = 6.1$ m, Top Tension = 350 kN, Effective Weight $w = 2.3$ kN/m, Semi-Restrained Top Condition, Current and Wave Cross with an Right Angle)	142
7.17	The Effect of Internal Flow on Maximum Displacement According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	144
7.18	The Effect of Internal Flow on Maximum Envelopes of a 152 m Riser According to the Change of Current Velocity under High Top Tension of 1550 kN. System is Same as Example of Fig. 7.16 except Parameters as Shown in This Figure	145
7.19	The Effect of Internal Flow on Maximum Envelopes of a 152 m Riser According to the Change of Current Velocity under Low Top Tension of 350 kN. System is Same as Example of Fig. 7.16 except Parameters as Shown in This Figure	146
7.20	The Effect of Internal Flow on Maximum Displacement According to the Increase of Wave Frequency. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	147
7.21	The Effect of Internal Flow on Maximum Envelopes of a 152 m Riser According to the Change of Wave Frequency under High Top Tension of 1550 kN. System is Same as Example of Fig. 7.16 except Parameters as Shown in This Figure	148
7.22	The Effect of Internal Flow on Maximum Envelopes of a 152 m Riser According to the Change of Wave Frequency under Low Top Tension of 350 kN. System is Same as Example of Fig. 7.16 except Parameters as Shown in This Figure	149
7.23	The Effect of Internal Flow on Maximum Displacement Fields According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	151
7.24	The Effect of Internal Flow on Maximum Displacement and Rotation Envelope Due to Current Loading for Three Types of Top Tension. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	152
7.25	The Effect of Internal Flow on Maximum Internal Stress Fields Due to Current According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	153
7.26	The Effect of Internal Flow on Maximum Shear and Bending Moment Envelope Due to Current Loading for Three Types of Top Tension. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	154

7.27	The Effect of Internal Flow on Maximum Displacement Fields Due to Vortex Shedding According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	156
7.28	The Effect of Internal Flow on Maximum Displacement and Rotation Envelopes Due to Vortex Shedding for Three Types of Top Tensions. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	157
7.29	The Effect of Internal Flow on Maximum Stress Fields Due to Vortex Shedding According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	158
7.30	The Effect of Internal Flow on Maximum Stress Envelopes Due to Vortex Shedding for Three Types of Top Tensions. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure	159

LIST OF TABLES

5.1	Comparison of System Frequencies of the 1st to 10 th Modes for a Drilling Riser System with No Internal Flow. (System frequency = ω_i (rad/sec), $V_1 = 0$ m/sec)	84
5.2	Comparison of System Frequencies of the 1st to 10 th Modes for a Drilling Riser System with Internal Flow. (System frequency = ω_i (rad/sec), $V_1 = 6$ m/sec)	85
6.1	Nodal Displacements and Rotations Due to Uniform Current in Transverse and Inline Direction (Time = 5 sec, Top Tension = 0, $w = 0$, Internal Velocity = 0)	95

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

THE EFFECT OF INTERNAL FLOW ON MARINE RISER DYNAMICS

By

NAMSEEG HONG

August 1994

Chairman: Dr. Robert G. Dean
Major Department: Coastal and Oceanographic Engineering

A mathematical model for the dynamic analysis of the riser system with the inclusion of internal flow and nonlinear effects due to large structural displacements is developed to investigate the effect of internal flow on marine riser dynamics. The riser system accounts for the nonlinear boundary conditions and includes a steady flow inside the pipe which is modeled as an extensible or inextensible, tubular beam subject to nonlinear three-dimensional hydrodynamic loads such as current or wave excitation. Combining Iwan-Blevin's model into the approximated form of the nonlinear model derived in this study, current-vortex model is derived to examine the effect of internal flow on vortex-induced vibration due to inline current.

A semi-analytical method using series expansion is employed to derive Eigenvalue problem to facilitate the evaluation of the system frequencies, and its validity is given through the comparison of the solutions with the conventional method using system matrices. Galerkin's finite element approximation and time-incremental operator are implemented to derive the matrix equation of equilibrium for the finite element system and the extensibility or inextensibility condition is used to reduce the degrees of freedom of

the system and the required computational time in the case of a nonlinear model. The algorithm is implemented to develop computer programs used in several numerical applications.

The investigations of the effect of internal flow on system frequency, vortex-induced vibration, and vibration due to current or wave loading are performed according to the change of various parameters such as top tension, internal flow velocity, current velocity, wave period, and so on. It is found that the effect of internal flow can be controlled by the increase of top tension. However, careful consideration has to be given in the design point, particularly for the long riser under the harmonic loading such as vortex-shedding or waves. And also, the consideration of nonlinear effects due to large structural displacements increases the effect of internal flow on riser dynamics.

CHAPTER 1 INTRODUCTION

1.1 General Description

The marine riser is a conductor pipe used in floating drilling operations to convey drilling fluid and to guide tools between the drilling vessel and the well head at the ocean floor. The riser can be a single rigid pipeline for the drilling riser system, or a bundle of flow lines that are assembled as an integral unit for the production riser system. At the surface, the riser system is connected to a surface vessel such as the tension leg platform (TLP), floating production and storage unit (FPSU). At the bottom, it may be hanging free or connected to a blow-out prevention (BOP) stack.

Risers are often used in the conventional sense to resist the applied forces and to transmit loads in the surface structure by its bending, shear and axial stiffness. To avoid the collapse and buckle of the riser by its own weight and to reduce the excess load on the manifold in the deep sea, a constant top tension is often applied. Buoyant material is strapped around the riser at intervals to provide an additional buoyancy and to reduce the required top tension. Various end-conditions for a riser system are possible, for example, hinged-hinged, fixed-free, etc. These are some of the features of the marine riser system.

The offshore industry is moving into deeper waters and more hostile environments. When exploratory drilling operations move into deeper water, the dynamic behaviors of riser should be more carefully considered because of the long free span. There are many factors that influence on the behavior of a riser, such as the offshore environment (wave and current), the behaviors of platform (movements or mean offsets), and the properties

of the riser itself (dimensions, weight, top tension, and internal flow). Most of these factors have been considered in earlier studies and many related papers have been published. However, it was not until the publication of a paper (Moe and Chucheepsakul, 1988) that the effect of internal flow on riser dynamics was investigated. Before Moe's investigation it was already known that the internal flow might cause a dynamic instability or buckling of a horizontal pipeline supported above ground and such effect has been considered in design and analysis of pipeline. Since the dynamics of pipe conveying fluid has been studied fairly extensively over the past 40 years, the basic techniques were recently adopted for the dynamic analysis of a marine riser with internal flowing fluid.

Figure 1.1 is a schematic of a marine riser conductor that contains internal fluid flow and is subject to environmental forces. When the internal fluid travels inside the curved path along the deflected riser, it experiences centrifugal and coriolis accelerations due, respectively, to the curvature of the riser and the relative motion of fluid to time dependent riser motion. Those accelerations exert against the riser which, in turn, affect the dynamic behavior of the riser and cause riser vibrations. In addition, the riser vibration could be "locked" with the flow-induced vortex shedding such that they resonate together to produce large deflections and stresses. This phenomenon could lead to the failure of the riser system prematurely.

In addition to this interaction problem, the riser behavior is also nonlinear. The nonlinearities are mainly of two origins that result from flow-induced drag force and from geometric nonlinearities due to large structural deflections. The latter becomes particularly significant for long risers over 1000 m. Such nonlinearities are due to large deflections and slopes, three-dimensional bending, extensibility, torsion, dependency of the hydrodynamic loads on the riser deformation and nonlinear boundary conditions. Three-dimensional bending is also important for such risers because the hydrodynamic loading due to the surface waves, the ocean current, the relative motion of the riser with

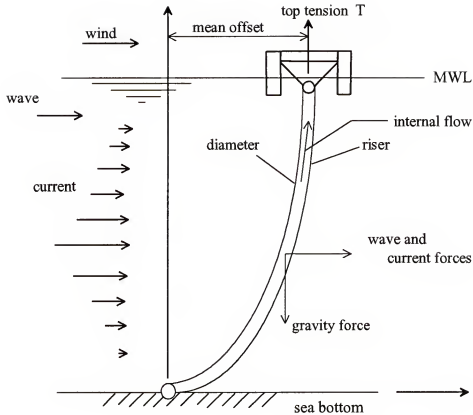


Figure 1.1: Configuration of Riser System with the Inclusion of Internal Flow

respect to water particle and the position of the supporting offshore platform, is actually three-dimensional. Extensibility effects are important for production risers made of flexible composite material. Torsional couples, concentrated at the upper end of the riser, arise from the variation of the relative orientation of the supporting offshore platform and the riser. The hydrodynamic loading exerted on the riser depends on the orientation of the riser tubes with respect to the surrounding flow and is deformation dependent. The displacements resulting from these nonlinear effects, in turn, modify the particle motion of the internal fluid. The system becomes a complicated interaction problem.

In summary, the primary objectives of this study are 1) to develop a mathematical model for the analysis of the riser system with the inclusion of internal flow and the aforementioned nonlinear effects, 2) to solve the model numerically, and 3) to examine the effect of the internal flow on riser dynamics. In this study, Iwan-Blevin's model (1990), which is one of the self-excited oscillator models, will be implemented as a vortex shedding model to examine the effect of the internal flow. Semi-analytical solution using series expansions and numerical methods using system matrices are both employed to derive the Eigenvalue problem to facilitate the evaluation of system's natural frequencies. For the solution of forced vibration, Galerkin's finite element approximation is implemented and the Blevins-Iwan model (1990) is adapted to solve vortex-induced vibration due to inline current. The effect of internal flow on system frequencies, vortex-induced vibration, wave-induced vibration, internal stresses as well as the kinematics of riser are investigated according to the change of internal flow or current velocity, water depth, top tension, wave period and so on.

1.2 Literature Review

The literature review contains three categories: 1) Instability of fluid-conveying pipe in which the basic concept is presented, 2) The effect of internal flow on riser dynamics in which the previous researches of this subject are discussed, and 3) Riser analysis in which rational ideas for extending previous works are given.

1.2.1 Instability of Fluid-Conveying Pipe

The vibration of pipe conveying fluid did not attract much attention until 1950 when severe vibrations of the Trans-Arabian pipeline were observed. Ashley and Haviland (1950) studied the bending vibrations of a pipeline in conjunction with the flow-induced vibrations of the Trans-Arabian pipeline. Housner (1952) was the first to correctly derive the governing equation of motion and predict instability. He concluded that the internal flow velocity might cause a dynamic coupling of the simple modes of

vibration so that the normal modes of vibration were of complex shape with 90- degree out of phase components. At low fluid velocities there was negligible effect on the vibration of the pipeline, but at a certain high critical velocity the fluid flow caused a dynamic instability. The induced centrifugal and coriolis accelerations become important in the deflected pipeline.

The type of instability depends on the end conditions of the pipe. Pipes supported at both ends blow out and buckle when the flow velocity exceeds the critical velocity (Housner, 1952; Homes, 1978). Benjamin (1961), dealing with the general dynamic problem of articulated pipe systems conveying fluid, on the other hand, showed that when such systems possess one free end, unstable oscillations may develop for sufficiently high velocities. Paidoussis (1966) demonstrated the existence of unstable oscillations of continuously flexible tubular cantilevers conveying fluid. In his study, a cylindrical body was immersed in fluid flowing parallel to the direction of internal flow and the forces exerted by the fluid, except those due to fluid friction, were caused from lateral acceleration. Later, he (Paidoussis, 1970) showed that cantilever pipes on the ground were flailed with large amplitude once the internal flow velocity exceeds a certain critical velocity.

Chen (1973) presented the out-of-plane motion of uniformly curved tubes containing flowing fluid. Hamilton's principle was used to derive both in-plane and out-of-plane equations of motion and a general solution for the natural frequency was obtained analytically. Hill and Davis (1974) dealt with the effect of initial forces, which could arise in a clamped tube as it is loaded by a pressurized flowing fluid on an initially planar curved tube. They obtained numerical solutions by using the finite element technique.

Paidoussis and Issid (1974) investigated the dynamics and instability of flexible pipes containing flowing fluid, where the flow velocity is either entirely constant, or with a small harmonic component superposed. Paidoussis (1986) re-examined the dynamics

and stability of short tubes by means of Timoshenko beam theory and a three-dimensional fluid model for the flow, rather than the plug-flow model commonly utilized. In their study, it was shown that for long tubes, Euler-Bernoulli beam theory and a plug-flow model were perfectly adequate.

In some of the latest works, Paidoussis (1988) and Blevins (1990) provided general reviews on dynamics of fluid-conveying pipes which listed many references about this subject.

1.2.2 The Effect of Internal Flow on Riser Dynamics

Although several studies have investigated the vibration of a pipeline conveying fluid supported above ground, there are only a few papers dealing with the effect of internal flow on marine riser dynamics.

Moe (1988) was the first who considered the forces due to the internal flowing fluid as a dynamically forcing component acting on the interior wall of the riser and derived the governing equation of motion. Before that, the loading due to the internal fluid was included in the internal tension of the riser as a fluid static force, that is, centrifugal and coriolis acceleration due to the internal flow were largely negligible.

Wu and Lou (1991) developed a mathematical model for the lateral bending vibration of a marine riser and examined the effect of internal flow and bending rigidity of the pipe on the dynamic behavior of the riser. Their mathematical model included the steady flow inside the pipe together with other factors such as currents or wave excitation and used a singular perturbation technique to solve it under the condition that the rigidity parameter, $\varepsilon = EI/L^3$, is sufficiently small for deep water risers. It was found from their results that the internal flow reduced the effect of top tension, but the riser motion was not significantly affected when the top tension of the riser was relatively high. However, the problem deserves further investigation since the system solved was linear and the perturbation technique was not valid for low top tension cases.

Chen (1992) derived the governing equation with the inclusion of dynamic force for lateral vibration by applying Hamilton's principle. The natural frequencies and the mode shapes were formulated and presented. The critical buckling and significant velocities which associate the internal flowing fluid with system integrity were also presented. In his paper, it was concluded that the natural frequencies could be reduced drastically by the fluid dynamic force if the tension was insufficient for a neutrally buoyant riser system, and that the dynamic force had less influence on a positively or negatively buoyant riser system. However, his system is also a linear model as was that of Wu and the method employed by him to estimate the natural frequencies and the modal shapes appeared to be quite approximate.

1.2.3 Riser Analysis

For the riser analysis without considering the effect of internal flow, considerable efforts have been made over the years. A marine riser can be treated as a long, slender tubular beam-column (Bernoulli-Euler type) connected at upper and lower ball joint. For linear analysis, several methods, e.g., finite difference, finite element method, or analytical method, etc., were used. Burke (1974) studied the dynamic riser response and discussed the effects of increasing water depth on riser design and operations. Dareing and Huang (1979) explored the application of modal analysis for calculating marine riser time-dependent stresses and suggested the simplification of dynamic response calculations, using only a limited number of eigenvalues and eigenfunctions. Chakrabati and Frampton (1982) presented a review on riser analysis techniques.

With increasing water depth, the nonlinear behaviors of the riser become more prominent as mentioned in section 1.1. The study of riser analysis including nonlinearities has been severely limited by the complex governing equations describing its equilibrium configuration. With the advances of numerical methods, however, it became feasible to solve nonlinear systems. The equations for large amplitude three-dimensional

inextensional motions of beams were derived by Nordgren (1974), with the assumption of constant principal moments of inertia, negligible rotatory inertia, and the uncoupled torsional motion. Bruce and Michael (1977) described a mathematical model and solution technique for a system with coupled dynamic axial and lateral responses of a riser column.

Fellipa and Chung (1981) formulated a mathematical model and implemented a finite element method for solving nonlinear static equilibrium configurations of deep water risers. Also, a transient analysis was developed by Chung (1981) for the determination of nonlinear motion by considering nonlinear static configuration as an initial condition for dynamic analysis. A tribute to their work was the consistent treatment of hydrodynamic loading.

Garrett (1982) presented a three-dimensional finite element model of an inextensible elastic rod with equal principal stiffness. The model permitted large deflections and finite rotations and accounted for tension variation along its length. Its use in static analysis was described and time integration method for dynamic analysis was developed.

Konuk (1982) provided a general foundation for developing rigorous formulation of problems involving marine pipelines with twist. A continuation technique was proposed to handle convergence problems.

Safai (1983) developed a method for automatically updating the structural geometry during the dynamic analysis for a system in which the bending, axial and torsional vibrations are uncoupled. The advantage of his method was that it could be applied to risers in great water depth with relatively large top displacements, for which the global rotation of each riser element easily exceeded the limit considered to discriminate large and small deformations, i.e. 8 degree.

Kim and Triantafyllou (1984) studied the nonlinear dynamics of long, slender cylinders assuming moderately large deformations and no longitudinal excitation modes. In

their paper, the nonlinearity in axial strain was represented through variable tension depending on time and space, while their bending equations were linear and uncoupled.

McNamara and Lane (1984) presented an efficient method based on the finite element approach using convected coordinates for arbitrary large rotations and established an associated computer program for computing the gross motions and internal forces of offshore columns and loading arms.

Huang and Chuchepsakul (1985) introduced a method of static analysis for risers experiencing large displacements. The method represented a modified functional involving multipliers to account for the arc length being unvaried with neglected tension, and used the exact curvature of radius in their formulation. This method is suitable for analyzing a riser having a known top tension and a possible slippage at the top slip joint.

Kokarakis and Bernitsas (1985) formulated the problem of static three-dimensional nonlinear, large deformation response of a riser within small strain theory and then solved it numerically by using an incremental finite element algorithm, which involved a prediction-correction scheme. The extensibility or inextensibility condition was used as the constitutive relation in the tangential direction. Torsion and bending were coupled. The external loads and boundary conditions were deformation dependent. Later, they (Bernitsas and Kokarakis, 1987) employed a previously formulated static model for the dynamic analysis of riser and developed an efficient algorithm for its numerical solution.

O'Brien and McNamara (1988) developed a technique based on finite element method by the separation of the rigid body motions from deformations of element under the condition of finite rotations. The method considered all the nonlinear effects of geometric changes, bending-axial, and bending-torsional coupling.

1.3 Scope of Study

As mentioned briefly earlier, the primary objective of this dissertation is to develop a mathematical model for the dynamic analysis of the riser system with the inclusion of

internal flow and nonlinear effects due to large structural displacements and solve it numerically for the investigation of the effect of internal flow on marine riser dynamics. Iwan-Blevin's model is implemented as a vortex-shedding model to examine the effect of internal flow on vortex-induced vibration due to inline current. A semi-analytical solution using series expansion is presented for obtaining system frequencies and a conventional numerical method using system matrices is employed to check the results with those from semi-analytical method. For the solution of forced vibration, Galerkin's finite element approximation is implemented and Iwan-Blevin's model is adapted to solve vortex-induced vibration.

Chapter 2 formulates the mathematical model of the system. The riser system is modeled as a long tubular beam with the inclusion of internal flow. First, the simplifications and assumptions made in this paper are described and the equations of dynamic equilibrium for the differential element of pipe are derived on the basis of Eulerian description for the removal of the complexities of the equations. The force acting on the interior wall of the riser, which results from the deformation-dependent accelerations due to internal flow, is derived in three ways and compared with each other. As a hydrodynamic external force, the normal force component to the deformed riser is developed using the concept of relative motion between the pipe and the surrounding fluid. Combining all the terms, the nonlinear governing equations are derived with nonlinear boundary conditions. The nonlinear governing equations and boundary conditions are approximated using the assumption of small deformation. Finally, the current-vortex model with the inclusion of internal flow is developed by combining the approximated forms with Iwan-Blevin's model.

Chapter 3 formulates the numerical model of governing equations and boundary conditions using finite element method. Variational statement for boundary value problem is introduced, that is, the weak form of governing equation is derived. Sequentially,

the incremental operator is applied on the weak form in order to derive the incremental equilibrium equation, and then Galerkin's decomposition method is performed to construct the approximate solutions of the problem. Finally, incorporation of the usual cubic shape functions yields the matrix dynamic equilibrium equations constructed in terms of the unknown deformations at the nodes. In addition, the finite element formulation for vortex-induced vibration is made in the last section using the same procedure except applying an incremental operator.

In chapter 4, the development of a finite element code for the solution of our two-point boundary value problem is discussed. Some considerations of code design are presented and the overall structure of the code is outlined.

The system frequencies and mode shapes of a marine riser with the internal flow are evaluated in chapter 5. This chapter consists of three categories; first, semi-analytical solutions are obtained using series expansion to the mathematical model and second, numerical solutions are calculated adapting complex eigenvalue method to the matrix equilibrium equation. Finally, the algorithm for the computer program is introduced and the verification of the first method is inspected through the comparison of the system frequencies with the second method. Further, the comparison of semi-analytical solutions with results from referenced papers is given.

Chapter 6 deals with the forced vibration of the system derived in chapter 3 and its dynamic behaviors. The system considered here consists of two; first, the system simply responding to the general environmental loads such as wave or current and second, the system responding to the vortex shedding generated in the transverse direction of inline current, that is, vortex-induced vibration. Newmark method is used for time step integration and predictor-corrector scheme is implemented as an iteration algorithm for the non-linear terms. Using those techniques mentioned above, the vortex-induced vibration due to current and inline vibration due to current or wave are obtained and then their

characteristics are investigated. Finally, the application of top boundary condition to the system matrices or vectors is discussed and its effect on the dynamic responses of riser is inspected.

In chapter 7, using all the programs developed in this study, the effects of internal flow on riser dynamics are investigated, that is, the effects on system frequencies, vortex-induced vibration, and vibrations due to current or wave loading by applying those programs and numerical models. The investigations are carried out in the way inspecting how the internal flow affects the kinematics or stresses of the riser with the various parameters. In addition, the characteristics of each forcing term due to internal flow, namely, centrifugal and coriolis terms, and the effects due to the inclusion of the geometric and the fluidic nonlinearity are discussed in this chapter.

Finally, chapter 8 includes the conclusions of this study with an investigative explanation and recommendations for further research.

CHAPTER 2 MATHEMATICAL MODEL OF THE SYSTEM

The riser system can be modeled as a long tubular beam connecting the drilling platform with the wellhead at the seabed and is composed of rigid pipes. Figure 2.1 depicts the simplified configuration of the riser system. The riser pipes are connected by the riser connectors. A ball joint is located at the lower end of the riser and a combination of a ball and a slip joint at the upper end. The ball joints alleviate high bending stresses and the slip joint compensates for the heave motion of the drilling platform. A tensioning system is installed on the drilling platform and applies a tension at the top of the riser. This tension provides part of the support required to keep the riser tight and prevent buckling or collapse. Additional supporting force is provided by buoyancy modules properly distributed along the riser. The kill and choke lines are small diameter pipes with high pressure, which run along the riser. The lines are mounted directly on the connectors through which they exert concentrated forces and moments of the riser.

The drill string is inside the riser, which runs from the drilling platform to the well. The ring space between the riser and the drill string circulates the drilling mud. It exerts on the riser static pressure force, coriolis and centrifugal forces due to the riser's local rotation, and vertical and torsional frictional forces. And also, the source of external forces exerted on the riser are the ocean currents, the surface and internal waves.

Due to the drifting of the platform and the variation of the current direction with depth, the centerline of the riser is in general deformed to a three-dimensional curve with the large deflections with respect to its original equilibrium position. The mathematical model is a set of consistent equations for the tubular beam with large three-dimensional

deflections under internal and external pressure. The extensional oscillations of the beam are also included in this formulation.

2.1 Simplifications and Assumptions of Model

The riser system can be modeled as described in Fig. 2.1. However, the inclusion of all the terms mentioned above makes the analysis extremely complicated. Therefore, the model derived in this chapter is based on the following simplifications which are usually made in the analysis of a long tubular beam.

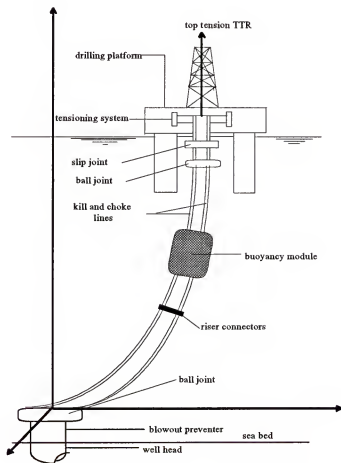


Figure 2.1: Simplified Configuration of the Riser System

1. The riser can be modeled as a beam rather than a shell because the ratio of its diameter to the riser length is small. If detailed analysis is necessary locally, the riser can be locally modeled as a shell. However, not only our knowledge of the distribution of hydrodynamic loads around cylinder is very limited but also detailed analysis is also complicated.

2. The drill string occasionally comes in contact with the riser through the tool joints and exerts concentrated forces and moments on the riser. The presence of the drill string is ignored. This will underestimate the weight of the riser.

3. The frictional force exerted on the riser due to the mud flow is not taken into account. Consequently, the plug-flow model with no radial variation of velocity is utilized.

4. The variation of temperature of water and drilling mud with depth, which may induce stresses, is neglected.

In addition to the foregoing simplifications of the riser system, the following assumptions are made.

1. The riser material is isotropic, homogeneous, elastic, and linear, i.e., it obeys Hook's law.

2. Plane sections remain plane after deformation, i.e., warping is neglected.

3. Cross sections remain normal to the neutral axis after deformation, i.e., shear deformation is neglected. Thus, the riser system can be modeled as a Bernoulli-Euler type of beam and not as a Timoshenko type.

4. The initial undeformed configuration of the riser is straight even though it is vertically inclined, i.e. initial imperfections are not present. Therefore, both initial curvatures and geometric torsion are zero, and adjacent cross sections of the riser are parallel to those in the undeformed state.

Based on these simplifications and assumptions, the model for a three-dimensional dynamic behavior of the riser is established. It is a set of consistent equations for beam with large deflections and consider the extensional oscillations of the riser. The hydrodynamic load due to wave and current is integrated over the external surface of the riser and the fluid force due to internal flow is exerted over the internal surface of the riser. The boundary conditions can be changed by slightly modifying the model and different initial conditions can be imposed.

2.2 Equations of the Pipe

This section derives the equations of dynamic equilibrium for a differential element of the pipe. The centerline of the riser is, in general, a three dimensional curve and Eulerian description and vector representations are applied for the removal of the complexities of the equations.

The main aspects of the curved geometry of the system are depicted in Figure 2.2. Three coordinates systems are defined, namely

- $(\hat{i}, \hat{j}, \hat{k})$, an orthonormal global inertial system of coordinates with origin at the lower ball joint of the riser.
- $(\hat{n}, \hat{b}, \hat{t})$, an orthonormal triad passing through the centroid of the cross section of the deformed configuration.
- $(\hat{N}, \hat{B}, \hat{T})$, an nonorthonormal triad passing through the centroid of the cross section of the deformed configuration.

where " $\hat{}$ " indicates a unit vector and hereafter boldface represents general vectors.

Since we assume no shear deformation, the third nonorthonormal triad becomes consistent with the second orthonormal triad. The initial undeformed configuration of the riser is vertically straight, i.e., its direction accords with $\hat{k}$. The deformed space curve is specified by giving its position vector $\mathbf{r}$ from the origin of space fixed system $(\hat{i}, \hat{j}, \hat{k})$ to a point on the deformed riser centerline (see Fig. 2.2), as a function of deformed

arclength s . At any point on the deformed curve the unit tangent vector $\hat{t}$, the unit normal vector $\hat{n}$, and the unit binomial vector $\hat{b}$ are defined by

$$\hat{t} = \mathbf{r}', \quad \hat{n} = \hat{t}' / \kappa, \quad \hat{b} = \hat{t} \times \hat{n} \quad (2.1)$$

where prime denotes differentiation with respect to s , and the curvature κ , from the well known results about the differential geometry of curves in space (Love, A., 1927), is given by

$$\kappa^2 = \mathbf{r}'' \cdot \mathbf{r}'' \quad \text{or} \quad -\mathbf{r}' \cdot \mathbf{r}''' \quad (2.2)$$

where " $\cdot$ " denotes inner product of vectors (dot product).

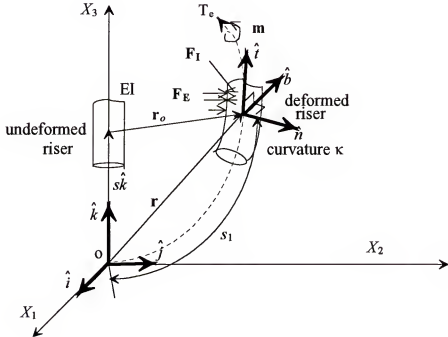


Figure 2.2: Position Vector and Principal Direction at a Point on the Riser Centerline

The total bending moment, as shown in Fig. 2.2, acts in the binomial direction and is proportional to the bending rigidity of the cross section EI , and the sign of curvature is fixed by convention since change of sign merely reverses the direction of $\hat{n}$ and $\hat{b}$ as seen from Eq. (2.1). For three dimensional curves, it is convenient to choose the positive sign so that $\hat{n}$ is in a fixed direction normal to the plane and $(\hat{n}, \hat{b}, \hat{t})$ form a right-handed orthonormal triad. Hereafter, the space curve will be identified with the central axis of the riser in the deformed state, with the position vector, as shown in Fig. 2.2, represented as follows.

$$\mathbf{r} = \mathbf{r}_0 + s\hat{k} = x_1\hat{i} + x_2\hat{j} + (s + x_3)\hat{k} \quad (2.3)$$

where $\mathbf{r}_0$ is the deformation vector at any point on the riser in the undeformed state.

In the classical theory of rods, the internal state of stress at a point on the rod is fully characterized by the resultant force $\mathbf{F}$ and the couple moment $\mathbf{M}$ acting on the central axis. Conservation of linear momentum leads to the equation of motion

$$\mathbf{F}' + \mathbf{F}_E + \mathbf{F}_I = m_r \ddot{\mathbf{r}} \quad (2.4)$$

where $\mathbf{F}_I$ and $\mathbf{F}_E$ are the internal fluid force and the applied external hydrodynamic force per unit length, respectively (see Fig 2.2) and m_r is the mass of the riser per unit length, and dot denotes differentiation with respect to time. Since the centerline of the riser is deformed to a three dimensional curve, the conservation of angular momentum must include the rotatory inertia even if the shear deformation is negligible. Thus the moment conservation equation becomes

$$\mathbf{M}' + \hat{t} \times \mathbf{F} + \mathbf{m} = [J_r] \{\ddot{\omega}\} \quad (2.5)$$

where $\mathbf{m}$ is the distributed couple vector induced by the asymmetric flow due to vortex shedding. It is usually negligible if we ignore the coupling of bending and torsion is weak. The mass moment of inertia $[J_r]$ is given by

$$[J_r] = \begin{bmatrix} J_{11} & 0 & 0 \\ 0 & J_{22} & 0 \\ 0 & 0 & J_{33} \end{bmatrix}$$

and $\dot{\omega}$ is the angular acceleration and is obtained from the time differentiation of the angular velocity. The angular velocity ω is computed using the appropriate transformation matrices that carry the initial system $(\hat{i}, \hat{j}, \hat{k})$ to the final triad $(\hat{n}, \hat{b}, \hat{t})$. Transformation from the initial system to the final triad takes place through three translations (x_1, x_2, x_3) and three rotations $(\theta_1, \theta_2, \theta_3)$ in the following sequence:

- rotation θ_3 about the $\hat{k}$ axis to get the intermediate system $(\hat{i}_1, \hat{j}_1, \hat{k}_1 = \hat{k})$
- rotation θ_2 about the new axis $\hat{j}_1$ to get the second intermediate system $(\hat{i}_2, \hat{j}_2 = \hat{j}_1, \hat{k}_2)$
- rotation θ_1 about the new axis $\hat{i}_2$ to get the final triad $(\hat{n}, \hat{b}, \hat{t})$

As a results, the angular velocity ω is given by

$$\omega = \dot{\theta}_3 \hat{k}_1 + \dot{\theta}_2 \hat{j}_2 + \dot{\theta}_1 \hat{n} \quad (2.6)$$

To perform the above rotations, we have to apply sequentially the following rotation matrices

$$[R_1] = \begin{bmatrix} \cos \theta_3 & \sin \theta_3 & 0 \\ -\sin \theta_3 & \cos \theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.7)$$

$$[R_2] = \begin{bmatrix} \cos \theta_2 & 0 & -\sin \theta_2 \\ 0 & 1 & 0 \\ \sin \theta_2 & 0 & \cos \theta_2 \end{bmatrix} \quad (2.8)$$

$$[R_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & \sin \theta_1 \\ 0 & -\sin \theta_1 & \cos \theta_1 \end{bmatrix} \quad (2.9)$$

Using Equations (2.7), (2.8) and (2.9), Eq. (2.6) can be presented in terms of the fixed inertial system

$$\begin{aligned} \boldsymbol{\omega} = & \left(\dot{\theta}_1 \cos \theta_2 \cos \theta_3 - \dot{\theta}_2 \sin \theta_3 \right) \hat{i} + \left(\dot{\theta}_1 \cos \theta_2 \sin \theta_3 + \dot{\theta}_2 \cos \theta_3 \right) \hat{j} \\ & + \left(\dot{\theta}_3 - \dot{\theta}_1 \sin \theta_2 \right) \hat{k} \end{aligned} \quad (2.10)$$

The angular acceleration $\dot{\boldsymbol{\omega}}$ can then be obtained by time differentiation of Eq. (2.10).

According to the classical Bernoulli-Euler theory of elastic rods with equal principal stiffness (Love, A.E.H., 1927), the bending component of the stress couple is proportional to the curvature κ and is directed along the binomial vector $\hat{b}$. In addition, the torsional component H of the stress couple is directed to the tangent vector $\hat{i}$. Thus, the constitutive equation for the stress couple can be written as

$$\mathbf{M} = EI\kappa\hat{b} + H\hat{i} \quad (2.11)$$

Upon substitution of Eq. (2.11) into Eq. (2.5) after elementary vector manipulations using Eq. (2.1), we have

$$\hat{i} \times \left[(EI\kappa\hat{n})' - H\kappa\hat{b} + \mathbf{F} \right] + H'\hat{i} + \mathbf{m} = [J_r] \{ \dot{\boldsymbol{\omega}} \} \quad (2.12)$$

The dot and cross product of $\hat{i}$ with Eq (2.12) using Eq. (2.2) yield respectively

$$H' + \hat{i} \cdot \mathbf{m} = \hat{i} \cdot ([J_r] \{ \dot{\boldsymbol{\omega}} \}) \quad (2.13)$$

$$\mathbf{F} = -(EI\mathbf{r}'')' + (T_e - EI\kappa^2)\mathbf{r}' + \mathbf{r}' \times \mathbf{m} + H\mathbf{r}' \times \mathbf{r}'' - \mathbf{r}' \times [J_r] \{ \dot{\boldsymbol{\omega}} \} \quad (2.14)$$

where T_e , the effective riser tension, is defined by

$$T_e = \hat{i} \cdot \mathbf{F} \quad (2.15)$$

Alternatively the effective riser tension, which is the tangential component of the internal force, may be given approximately by (Dareing, 1976)

$$T_e \approx T + \rho_w g \pi D_o^2 (S_w - z)/4 - \rho_m g \pi D_i^2 (S_m - z)/4 \quad (2.16)$$

where T = actual tension in the riser, S_w = ordinate of the free surface of the water, S_m = ordinate of the free surface of mud, ρ_w and ρ_m are the densities of water and mud respectively, D_o and D_i = external and internal diameter of the riser and z is measure from the lower ball joint, i.e., $z = s + x_3$.

The actual tension T also satisfies the constitutive relation:

$$T = EA \epsilon_t \quad (2.17)$$

where EA is the stretching rigidity and ϵ_t is the strain of the riser centerline in the tangential direction defined as

$$\epsilon_t = \left[\frac{dr}{ds} \cdot \frac{dr}{ds} \right]^{\frac{1}{2}} - 1 \quad (2.18)$$

where s is related to s_1 by

$$\frac{ds_1}{ds} = 1 + \epsilon_t \quad \text{or} \quad \frac{ds}{ds_1} = \frac{1}{1 + \epsilon_t} \quad (2.19)$$

For an inextensible riser, $\epsilon_t = 0$ and the deformed arclength s_1 becomes the undeformed arclength s from Eq. (2.19). However, Eq. (2.16) and Eq. (2.17) have to be kept to compute ϵ_t for the extensible riser.

Equation (2.13) indicates the torsional relation. If no distributed torsional couple is applied to the riser and a rotatory inertia is neglected, as shown in Eq. (2.13), H is independent of s_1 just as in the static theory of rods with equal principal stiffnesses. If a distributed torsional couple acts or a rotatory couple is included, then H can be determined by Eq. (2.13) and the prescribed value of H at one end.

Substituting Eq. (2.14) into Eq. (2.4), we can obtain the vector equation of motion for the riser

$$-(EI\mathbf{r}'')'' + [(T_e - EI\kappa^2)\mathbf{r}']' + (\mathbf{r}' \times \mathbf{m})' + (H\mathbf{r}' \times \mathbf{r}'')' - (\mathbf{r}' \times [\mathbf{J}_r] \{\dot{\omega}\})' + \mathbf{F}_E + \mathbf{F}_I = m_r \ddot{\mathbf{r}} \quad (2.20)$$

In this equation, the structural damping term is not considered. The effective tension T_e is unknown and $\mathbf{r}$ must also satisfy the extensibility or the inextensibility condition as discussed earlier.

2.3 Force due to Internal Flow

In this section, the force acting on the internal wall of the riser is derived in three ways: by obtaining the acceleration of a fluid particle, by using the concept of Hamilton's principle, and by evaluating the forcing components. In all three ways, no small-scale motions such as turbulence or secondary flow are assumed to be absent. And also, the plug-flow model with no radial variation of velocity is utilized as a fluid model for the internal flow as discussed in section 2.1.

First, we obtain the acceleration of a fluid particle by differentiating the fluid velocity $\mathbf{V}_I$,

$$\mathbf{A}_I = \frac{d\mathbf{V}_I}{dt} = \frac{\partial \mathbf{V}_I}{\partial s_1} \frac{ds}{dt} + \frac{\partial \mathbf{V}_I}{\partial t} \quad (2.21)$$

where

$$\mathbf{V}_I = \frac{\partial x_1}{\partial t} \hat{i} + \frac{\partial x_2}{\partial t} \hat{j} + \frac{\partial x_3}{\partial t} \hat{k} + V_1 \hat{t} = \dot{\mathbf{r}} + V_1 \mathbf{r}' \quad (2.22)$$

Performing the operations in Eq. (2.21) with $s_1 \approx s$ and $ds/dt = V_1$, we obtain the fluid acceleration in the deformed direction

$$\begin{aligned} \mathbf{A}_I &= (\dot{\mathbf{r}} + V_1 \mathbf{r}')' V_1 + \dot{V}_1 \mathbf{r}' \\ &= \ddot{\mathbf{r}} + \dot{V}_1 \mathbf{r}' + 2V_1 \dot{\mathbf{r}}' + V_1 V_1' \mathbf{r}' + V_1^2 \mathbf{r}'' \end{aligned} \quad (2.23)$$

Assuming steady flow and no convective acceleration along the riser curve, the fluid force acting on the internal wall of riser becomes

$$\mathbf{F}_I = -m_f \mathbf{A}_I = -m_f (\ddot{\mathbf{r}} + 2V_I \dot{\mathbf{r}}' + V_I^2 \mathbf{r}'') \quad (2.24)$$

Second, we apply the concept of Hamilton's principle just considering the finite part comprising the pipe and the enclosed volume of fluid. Hamilton's principle in our problem states that

$$\delta \int_{t_1}^{t_2} (T_r + T_f - V_r - V_f) dt = 0 \quad (2.25)$$

where, T_r and V_r are the kinetic and potential energies associated with the tube, and T_f and V_f are the corresponding quantities for the fluid. Using the velocity of the internal fluid obtained from neglecting the stretching strain in Eq. (2.22), we have kinetic energy of the enclosed volume of fluid

$$T_f = \frac{m_f}{2} \int_{t_1}^t \left[V_I^2 + \left(\frac{\partial x_1}{\partial t} + V_I \frac{\partial x_1}{\partial s_1} \right)^2 + \left(\frac{\partial x_2}{\partial t} + V_I \frac{\partial x_2}{\partial s_1} \right)^2 + \left(\frac{\partial x_3}{\partial t} + V_I \frac{\partial x_3}{\partial s_1} + 1 \right)^2 \right] ds_1 \quad (2.26)$$

and the potential energy of the fluid is zero because the fluid is assumed to be incompressible, i.e.,

$$V_f = 0 \quad (2.27)$$

Performing the variation after the substitution of Eq. (2.26) and (2.27) into (2.25), we obtain the following integration

$$\begin{aligned} \int_{t_1}^{t_2} \int_I [m_f \{ (\dot{x}_1 + V_I x'_1)(\delta \dot{x}_1 + V_I \delta x'_1) + (\dot{x}_2 + V_I x'_2)(\delta \dot{x}_2 + V_I \delta x'_2) \\ + (\dot{x}_3 + V_I x'_3 + 1)(\delta \dot{x}_3 + V_I \delta x'_3) \} + \delta T_r - \delta V_r] ds_1 dt \end{aligned} \quad (2.28)$$

$$\text{Since} \quad \delta \dot{x}_i = \frac{\partial}{\partial t}(\delta x_i), \quad \delta x'_i = \frac{\partial}{\partial x_i}(\delta x_i) \quad (i = 1, 2, 3)$$

each terms may be integrated by parts so as to eliminate the various derivatives of δx_i . When this is done, there is obtained

$$\int_{t_1}^{t_2} \int_l \left[\sum_{i=1}^3 (m_f \ddot{x}_i + 2m_f V_i \dot{x}_i' + m_f V_i^2 x_i'') \delta x_i + \delta T_r - \delta V_r \right] ds_1 dt = 0 \quad (2.29)$$

where all the integrated terms have disappeared because of the boundary conditions. From the concept of the resulting Euler-Lagrange equations, we can recognize that the expression in round bracket represents the forcing components of a fluid particle inside the pipe due to internal flow. Thus, the fluid force acting on the internal wall of the pipe can be written in vector form

$$\mathbf{F}_I = -m_f(\ddot{\mathbf{r}} + 2\mathbf{V}_I \dot{\mathbf{r}}' + V_I^2 \mathbf{r}'') \quad (2.30)$$

We can easily see that Equations (2.24) and (2.30) are identical. The first term on the right represents the inertia force associated with the riser acceleration. The second term is the inertia force associated with the coriolis acceleration which arise because the fluid is flowing with velocity V_I relative to the riser, while the riser itself has an angular velocity at any point along its length. The last term represents the inertia force associated with the change in direction of the flow velocity, owing to the curvature of the riser.

Finally, we can evaluate all the forcing components in a physical sense. There are three kinds of inertia forces by the drilling fluid: the force due to linear acceleration of the riser, the coriolis force, and the centrifugal force. From the general knowledge of those components (Hibbler, R.C., 1986), they are given by Eq. (2.31) to (2.33), respectively.

$$\mathbf{F}_{Ila} = m_f \ddot{\mathbf{r}} \quad (2.31)$$

$$\mathbf{F}_{Ico} = 2m_f \boldsymbol{\omega} \times \mathbf{V}_I \quad (2.32)$$

$$\mathbf{F}_{Ice} = \boldsymbol{\omega}_c \times (\boldsymbol{\omega} \times \hat{n}/\kappa) \quad (2.33)$$

where $\boldsymbol{\omega}_c = V_I \kappa \hat{n}$ and $\mathbf{V}_I = V_I \hat{t}$, and also $\boldsymbol{\omega}_c$ can be calculated from Eq. (2.10).

2.4 Hydrodynamic Forces

The relative motion between the pipe and the surrounding fluid produces hydrodynamic force composed of inertia, drag, and frictional forces. The resultant total force distribution $\mathbf{F}_E$ along riser length is decomposed into a normal force component, $\mathbf{F}_N$, and a tangential component, $\mathbf{F}_T$. To perform this decomposition, the relative velocity and acceleration is resolved into components perpendicular and parallel to the deformed riser axis as shown in Figure 2.3. The relative velocity vector between the pipe and the surrounding fluid is given by

$$\mathbf{V}_R = \mathbf{V}_P - \mathbf{V}_C - \mathbf{V}_W = \dot{\mathbf{r}} - \mathbf{V}_C - \mathbf{V}_W \quad (2.34)$$

where $\mathbf{V}_P$ = pipe velocity vector, $\mathbf{V}_C$ = steady current velocity vector, and $\mathbf{V}_W$ = time dependent wave velocity vector. It should be noted that current and wave velocity are functions of vertical position. Figure 2.3 depicts the velocity vectors acting on a segment of pipe and explains the definition of the relative velocity components. Noting that the current velocity is independent of time, the relative acceleration of the pipe with respect to the fluid is given by

$$\dot{\mathbf{V}}_R = \dot{\mathbf{V}}_P - \dot{\mathbf{V}}_W = \ddot{\mathbf{r}} - \dot{\mathbf{V}}_W \quad (2.35)$$

Pipe velocity:	$\mathbf{V}_P = \dot{\mathbf{r}}$
Fluid velocity (current + waves):	$\mathbf{V}_F$
Relative velocity:	$\mathbf{V}_R = \dot{\mathbf{r}} - \mathbf{V}_F$
Pipe tangent vector:	$\hat{\mathbf{i}}$
Tangential component of $\mathbf{V}_R$:	$\mathbf{V}_{RT} = (\mathbf{V}_R \cdot \hat{\mathbf{i}})\hat{\mathbf{i}}$
Normal component of $\mathbf{V}_R$:	$\mathbf{V}_{RN} = \mathbf{V}_R - \mathbf{V}_{RT}$

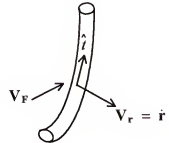


Figure 2.3: Velocity Vectors Acting on a Segment of Pipe and Definition of the Relative Velocity Components

and its tangential and normal components are given by

$$\dot{\mathbf{V}}_{\text{RT}} = (\dot{\mathbf{V}}_{\text{R}} \cdot \hat{\mathbf{i}}) \hat{\mathbf{i}} \quad \text{and} \quad \dot{\mathbf{V}}_{\text{RN}} = \dot{\mathbf{V}}_{\text{R}} - \dot{\mathbf{V}}_{\text{RT}} \quad (2.36)$$

Also, we need the tangential and normal components of the wave-induced water particle acceleration to calculate wave induced force and they are given by

$$\dot{\mathbf{V}}_{\text{WT}} = (\dot{\mathbf{V}}_{\text{W}} \cdot \hat{\mathbf{i}}) \hat{\mathbf{i}} \quad \text{and} \quad \dot{\mathbf{V}}_{\text{WN}} = \dot{\mathbf{V}}_{\text{W}} - \dot{\mathbf{V}}_{\text{WT}} \quad (2.37)$$

The normal component of the relative velocity, the current velocity, and the wave velocity vectors are expressed as a component form in x_1, x_2, x_3 directions.

$$\begin{aligned} \mathbf{V}_{\text{RN}} &= V_{\text{RN1}} \hat{\mathbf{i}} + V_{\text{RN2}} \hat{\mathbf{j}} + V_{\text{RN3}} \hat{\mathbf{k}} \\ \mathbf{V}_{\text{C}} &= u_{\text{c}} \hat{\mathbf{i}} + v_{\text{c}} \hat{\mathbf{j}} + w_{\text{c}} \hat{\mathbf{k}} \\ \mathbf{V}_{\text{W}} &= u_{\text{w}} \hat{\mathbf{i}} + v_{\text{w}} \hat{\mathbf{j}} + w_{\text{w}} \hat{\mathbf{k}} \end{aligned} \quad (2.38)$$

where,

$$\begin{aligned} V_{\text{RN1}} &= \dot{x}_1 - u_{\text{c}} - u_{\text{w}} - [x'_1 (\dot{x}_1 - u_{\text{c}} - u_{\text{w}}) + x'_2 (\dot{x}_2 - v_{\text{c}} - v_{\text{w}}) \\ &\quad + (x'_3 + 1/(1 + \varepsilon_t)) (\dot{x}_3 - w_{\text{c}} - w_{\text{w}})] x'_1 \\ V_{\text{RN2}} &= \dot{x}_2 - v_{\text{c}} - v_{\text{w}} - [x'_1 (\dot{x}_1 - u_{\text{c}} - u_{\text{w}}) + x'_2 (\dot{x}_2 - v_{\text{c}} - v_{\text{w}}) \\ &\quad + (x'_3 + 1/(1 + \varepsilon_t)) (\dot{x}_3 - w_{\text{c}} - w_{\text{w}})] x'_2 \\ V_{\text{RN3}} &= \dot{x}_3 - w_{\text{c}} - w_{\text{w}} - [x'_1 (\dot{x}_1 - u_{\text{c}} - u_{\text{w}}) + x'_2 (\dot{x}_2 - v_{\text{c}} - v_{\text{w}}) \\ &\quad + (x'_3 + 1/(1 + \varepsilon_t)) (\dot{x}_3 - w_{\text{c}} - w_{\text{w}})] (x'_3 + 1/(1 + \varepsilon_t)) \end{aligned} \quad (2.39)$$

and the normal component of the relative acceleration and the wave velocity vectors as a in directional components

$$\begin{aligned} \dot{\mathbf{V}}_{\text{RN}} &= \dot{V}_{\text{RN1}} \hat{\mathbf{i}} + \dot{V}_{\text{RN2}} \hat{\mathbf{j}} + \dot{V}_{\text{RN3}} \hat{\mathbf{k}} \\ \dot{\mathbf{V}}_{\text{W}} &= \dot{u}_{\text{w}} \hat{\mathbf{i}} + \dot{v}_{\text{w}} \hat{\mathbf{j}} + \dot{w}_{\text{w}} \hat{\mathbf{k}} \end{aligned} \quad (2.40)$$

where,

$$\begin{aligned}
 \dot{V}_{RN1} &= \ddot{x}_1 - \dot{u}_w - [x'_1 (\ddot{x}_1 - \dot{u}_w) + x'_2 (\ddot{x}_2 - \dot{v}_w) + (x'_3 + 1/(1 + \varepsilon_t))(\ddot{x}_3 - \dot{w}_w)] x'_1 \\
 \dot{V}_{RN2} &= \ddot{x}_2 - \dot{v}_w - [x'_1 (\ddot{x}_1 - \dot{u}_w) + x'_2 (\ddot{x}_2 - \dot{v}_w) + (x'_3 + 1/(1 + \varepsilon_t))(\ddot{x}_3 - \dot{w}_w)] x'_2 \\
 \dot{V}_{RN3} &= \ddot{x}_3 - \dot{w}_w - [x'_1 (\ddot{x}_1 - \dot{u}_w) + x'_2 (\ddot{x}_2 - \dot{v}_w) \\
 &\quad + (x'_3 + 1/(1 + \varepsilon_t))(\ddot{x}_3 - \dot{w}_w)] (x'_3 + 1/(1 + \varepsilon_t)) \quad (2.41)
 \end{aligned}$$

Finally, the normal component of the water particle acceleration of wave is given by

$$\dot{V}_{WN} = \dot{V}_{WN1} \hat{i} + \dot{V}_{WN2} \hat{j} + \dot{V}_{WN3} \hat{k} \quad (2.42)$$

where,

$$\begin{aligned}
 \dot{V}_{WN1} &= \dot{u}_w - [x'_1 \dot{u}_w + x'_2 \dot{v}_w + (x'_3 + 1/(1 + \varepsilon_t)) \dot{w}_w] x'_1 \\
 \dot{V}_{WN2} &= \dot{v}_w - [x'_1 \dot{u}_w + x'_2 \dot{v}_w + (x'_3 + 1/(1 + \varepsilon_t)) \dot{w}_w] x'_2 \\
 \dot{V}_{WN3} &= \dot{w}_w - [x'_1 \dot{u}_w + x'_2 \dot{v}_w + (x'_3 + 1/(1 + \varepsilon_t)) \dot{w}_w] (x'_3 + 1/(1 + \varepsilon_t)) \quad (2.43)
 \end{aligned}$$

the wave or particle velocity and acceleration are obtained from the velocity potential approach.

In the present case, the tangential force component by skin friction is neglected, and only the normal force component is retained by

$$\begin{aligned}
 \mathbf{F}_E \approx \mathbf{F}_N &= -\rho_w (\pi D_0^2/4) C_A \dot{V}_{RN} \\
 &\quad + \rho_w (\pi D_0^2/4) \dot{V}_{WN} - \rho_w D_0 C_D |\mathbf{V}_{RN}| \mathbf{V}_{RN}/2 \quad (2.44)
 \end{aligned}$$

where $|\mathbf{V}_{RN}| = [V_{RN1}^2 + V_{RN2}^2 + V_{RN3}^2]^{1/2}$ and C_A = added mass coefficient.

The first term of Eq. (2.44) is an inertia term representing the force that is required to accelerate the pipe with respect to the surrounding water. The second term is the wave induced force. This force is produced by the local pressure gradient that accompanies the normal component of water particle acceleration. The last term is the drag force that is

proportional to the square of the normal velocity component and is formed by the separation of flow.

2.5 Equations of Motion and Boundary Conditions

2.5.1 Nonlinear Governing Equations

Eq. (2.20) is the bending equations of motion of the pipe. In this equation, H is the time dependent torque due to variation of the relative orientation of the supporting platform and imperfections of the tensioning system, and $\mathbf{m}$ is the distributed couple vector induced by the asymmetry of the flow due to vortex shedding. But $\mathbf{m}$ is usually negligible, leading to the uncoupling of the bending and torsion. Furthermore, rotatory inertia term can be neglected because the riser response is usually dominated by low frequency motion. Thus, one deduces from Eq. (2.13) that H is independent of the pipe length, that is, it has constant value along the riser arclength. Eq. (2.20) then becomes

$$-(EI\mathbf{r}'')'' + [(T_e - EI\kappa^2)\mathbf{r}']' + H(\mathbf{r}' \times \mathbf{r}'')' + \mathbf{F}_E + \mathbf{F}_I = m_r \ddot{\mathbf{r}} \quad (2.45)$$

Substituting Eq. (2.30) and (2.44) into the above equation

$$\begin{aligned} m_r \ddot{\mathbf{r}} + (EI\mathbf{r}'')'' - [(T_e - EI\kappa^2)\mathbf{r}']' - H(\mathbf{r}' \times \mathbf{r}'')' \\ = -m_f(\ddot{\mathbf{r}} + 2V_I \dot{\mathbf{r}}' + V_I^2 \mathbf{r}'') \\ - \rho_w(\pi D_0^2/4)C_A \dot{\mathbf{V}}_{RN} + \rho_w(\pi D_0^2/4)\dot{\mathbf{V}}_{WN} - \rho_w D_0 C_D |\mathbf{V}_{RN}| \mathbf{V}_{RN}/2 \end{aligned} \quad (2.46)$$

Manipulating and rewriting the last three terms of right hand side after the substitution of Eq. (2.36) and (2.37) into the last three terms of (2.46)

$$\begin{aligned} -\frac{1}{2}\rho_w C_D D_0 |\mathbf{V}_{RN}| \mathbf{V}_{RN} - \rho_w A_o C_A \ddot{\mathbf{r}} + \rho_w A_o (1+C_A) \dot{\mathbf{V}}_W \\ + \rho_w A_o C_A (\hat{t} \cdot \ddot{\mathbf{r}}) \hat{t} - \rho_w A_o (1+C_A) (\hat{t} \cdot \dot{\mathbf{V}}_W) \hat{t} \end{aligned} \quad (2.47)$$

where $A_o = \pi D_0^2/4$

Substituting Eq. (2.47) into (2.46), and letting

$$\begin{aligned} \mathbf{q} = & -\frac{1}{2}\rho_w C_D D_o |\mathbf{V}_{RN}| \mathbf{V}_{RN} + \rho_w A_o (1+C_A) \dot{\mathbf{V}}_w \\ & + \rho_w A_o C_A (\hat{t} \cdot \hat{\mathbf{r}}) \hat{t} - \rho_w A_o (1+C_A) (\hat{t} \cdot \dot{\mathbf{V}}_w) \hat{t} \end{aligned}$$

$$\text{and} \quad m_t = m_r + m_f + \rho_w A_o C_A \quad (2.48)$$

we have nonlinear governing equations (N.G.E.'s).

$$\begin{aligned} m_t \ddot{\mathbf{r}} + 2m_f V_1 \dot{\mathbf{r}}' + m_f V_1^2 \mathbf{r}'' + (EI \mathbf{r}'')'' \\ - [(T_e - EI \kappa^2) \mathbf{r}']' - H(\mathbf{r}' \times \mathbf{r}'')' = \mathbf{q} \end{aligned} \quad (2.49)$$

The effective tension needed in the above equation can be computed in two ways: first by using Eq. (2.15) and unknown internal force $\mathbf{F}$ found from the local equilibrium equation which will be derived in next chapter and second by using Eqs. (2.16) and (2.17). Actually, the first method should be used to evaluate T_e and the second to compute ε_t for extensible risers. For inextensible risers, $\varepsilon_t = 0$ must be used instead of Eq. (2.17).

Utilization of the kinematic constraint on the unit tangent vector, $|\hat{t}| = |\mathbf{r}'| = 1$, leads to the reduction of number of the unknowns. The following kinematic relations are produced from this constraint.

$$x'_3 = (1 - x_1'^2 - x_2'^2)^{1/2} - 1/(1 + \varepsilon_t) \quad (2.50)$$

$$x_3(s_1) = x_3(0) + \int_0^{s_1} x'_3 ds_1 \quad (2.51)$$

Consequently, the vertical deflections can be computed in terms of lateral one. The kinematic relations are used as predictors to remove the vertical unknowns from the governing equation. They are used again as correctors, after the solution of the reduced equations has been obtained. This will reduce the iteration or time steps.

Also, the derivative of the position vector with respect to time and deformed arclength can be represented by

$$\begin{aligned}
 \hat{t} &= \mathbf{r}' = x'_1 \hat{i} + x'_2 \hat{j} + (x'_3 + 1/(1 + \varepsilon_t)) \hat{k} \\
 &= x'_1 \hat{i} + x'_2 \hat{j} + (1 - x_1'^2 - x_2'^2)^{1/2} \hat{k} \\
 \mathbf{r}' &= \dot{x}'_1 \hat{i} + \dot{x}'_2 \hat{j} - (1 - x_1'^2 - x_2'^2)^{-1/2} (x'_1 \dot{x}'_1 + x'_2 \dot{x}'_2) \hat{k} \\
 \mathbf{r}'' &= x''_1 \hat{i} + x''_2 \hat{j} - (1 - x_1'^2 - x_2'^2)^{-1/2} (x'_1 x''_1 + x'_2 x''_2) \hat{k} \\
 \kappa &= |\mathbf{r}''| = \{x_1''^2 + x_2''^2 - (1 - x_1'^2 - x_2'^2)^{-1} (x'_1 x''_1 + x'_2 x''_2)^2\}^{1/2} \\
 \mathbf{r} &= \ddot{x}_1 \hat{i} + \ddot{x}_2 \hat{j} + \ddot{x}_3 \hat{k}
 \end{aligned} \tag{2.52}$$

2.5.2 Boundary Conditions

For the completion of the mathematical model, the boundary conditions have to be specified. The boundary conditions for the dynamic model of riser are usually time dependent because of the motions of the supporting platform. The riser support can be modeled by substituting the linear translational or rotational springs providing the restoring boundary force and moments. Typically the displacement or the force vector and the unit tangent vector, or the bending or torsional moment has to be specified at each riser end. Figure 2.4 describes the free body diagram about the equilibrium of forces for the differential element at the top of the riser. From the free body diagram, equilibrium of forces at the top yields:

$$F_{1u} = (TF) \hat{t} \cdot \hat{i} - K_1 x_{1u} \tag{2.53}$$

$$F_{2u} = (TF) \hat{t} \cdot \hat{j} - K_2 x_{2u} \tag{2.54}$$

$$F_{3u} = (TF) \hat{t} \cdot \hat{k} - K_3 x_{3u} + TTR \tag{2.55}$$

where

$$TF \approx \rho_w g \pi D_0^2 (S_w - z)/4 - \rho_m g \pi D_i^2 (S_m - z)/4 \tag{2.56}$$

the subscript u indicates the upper end of the riser, TTR is the tension applied at top of the riser by the tensioning system and K_1 , K_2 , K_3 are spring constants supplied by the restoring boundary force. Equations (2.53) - (2.55) show that the fluidic tension at the top of the riser always acts in the tangential direction. Both TF and its direction are deformation dependent. Further, the internal force $\mathbf{F}$ is also deformation dependent. The nonlinearity of these boundary conditions disappear in linear model because the three direction cosines in (2.53) - (2.55) become 0,0, and 1 respectively.

The fixed-hinge support are applied at the lower end of the riser as boundary conditions. The reaction force vector or zero displacement vector and zero moment or the unit tangent vector are given, and the torsional moment at the lower or upper end.

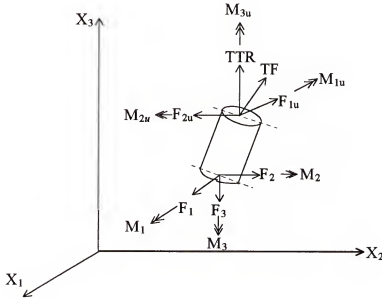


Figure 2.4: Free Body Diagram of the Equilibrium of Forces and Moments for the Differential Element at the Top of the Riser

2.5.3 Approximate Governing Equations and B.C.s

In two previous sections, the geometric nonlinearities resulted from large deflection and rotation have been considered in the derivation of the governing equations and boundary conditions. Eulerian description has been implemented to remove the complexity of the equations and the differentiation with respect to the deformed arclength have been operated in their expressions. However, it is no longer necessary for the analysis of the riser in an intermediate water depth and we can utilize an assumption of the small deflection beam theory.

In the riser problem in intermediate water depth, the deformed arclength s_f can be approximately identified with the undeformed arclength s for inextensible riser and the nonlinear vector equation (2.20) can be used except the fact that prime denotes differentiation with respect to s (Nordgren, R.P., 1974). Further, if we notify the approximation used to obtain the fluid acceleration due to internal flow (between Eqs. (2.22) and (2.23)), we can have the following equation:

$$\begin{aligned} m_t \ddot{\mathbf{r}} + 2m_f V_1 \dot{\mathbf{r}}' + m_f V_1^2 \mathbf{r}'' + (EI \mathbf{r}'')'' \\ - [(\mathbf{T}_e - EI \kappa^2) \mathbf{r}']' - H(\mathbf{r}' \times \mathbf{r}'')' = \mathbf{q} \end{aligned} \quad (2.57)$$

where prime denotes the differentiation with respect to the undeformed arclength s .

For the extensible riser, the actual tension is given by Hook's law as

$$\mathbf{T} = EA \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t = (\mathbf{r}' \cdot \mathbf{r}')^{1/2} - 1 \quad (2.58)$$

For nearly vertical riser, we introduce a rectangular Cartesian coordinate system with the x_3 axis vertically straight upward. And then, with small deflection and rotation, i.e., $x_1', x_2' \ll 1$ and $s + x_3 \approx s$, the curvature and the strain along centerline are given by

$$\begin{aligned} \kappa &= [\{x_1''(1 + x_1'^2)^{-3/2}\}^2 + \{x_2''(1 + x_2'^2)^{-3/2}\}^2]^{1/2} \\ &\approx (x_1''^2 + x_2''^2)^{1/2} \end{aligned} \quad (2.59)$$

$$\varepsilon_t = x_3' + (x_1'^2 + x_2'^2)/2 \quad (2.60)$$

In the above equation, the nonlinear terms in bracket have been retained in order to include the effect of lateral deflection on tensions.

Restricting attention to risers whose deformation lies wholly in a single plane and dropping nonlinear terms, the vector equation (2.57) reduces to

$$m_i \ddot{x}_1 + 2m_f V_1 \dot{x}_1' + m_f V_1^2 x_1'' + (E L x_1'')'' - (T_e x_1')' + H x_2''' = q_1 \quad (2.61)$$

$$m_i \ddot{x}_2 + 2m_f V_1 \dot{x}_2' + m_f V_1^2 x_2'' + (E L x_2'')'' - (T_e x_2')' - H x_1''' = q_2 \quad (2.62)$$

$$m_i \ddot{x}_3 - T_e' = q_3 \quad (2.63)$$

Most riser problems in intermediate water depth, longitudinal vibration is unimportant and we may neglect the longitudinal inertia term in Eq. (2.63). Further, it is convenient to include the hydrostatic effects of internal and external fluid pressures by defining effective weight per unit length w and effective tension T_e as

$$w = w_r + \gamma_i A_i - \gamma_o A_o \quad (2.64)$$

$$T_e = T - p_i A_i + p_o A_o \quad (2.65)$$

where w_r = riser weight per unit length, γ_i, γ_o = specific weight of internal and external fluid, p_i, p_o = internal and external static pressure of riser, and A_i, A_o = internal and external area of riser.

Eq. (2.64) leads to $q_3 = -w$. Eq. (2.63) can then be integrated to give

$$T_e = TTR - \int_0^L w ds \quad (2.66)$$

or

$$T_e = TTB + \int_0^L w ds \quad (2.67)$$

where TTR and TTB are, respectively, the top tension and the bottom reaction tension.

Thus, the effective tension is independent of the horizontal deflections and then, with no torsion, Eqs. (2.61) and (2.62) becomes

$$m_i \ddot{x}_1 + 2m_f V_1 \dot{x}_1' + m_f V_1^2 x_1'' + (ELx_1'')'' - wx_1' - T_e x_1'' = q_1 \quad (2.68)$$

$$m_i \ddot{x}_2 + 2m_f V_1 \dot{x}_2' + m_f V_1^2 x_2'' + (ELx_2'')'' - wx_2' - T_e x_2'' = q_2 \quad (2.69)$$

Equations (2.68) and (2.69) represent the linear model and can be also derived by applying Hamilton's principle (Chen, B.C.M., 1992). Detailed are given in Appendix A.

The boundary conditions associated with linear model can be found by modifying the direction cosines of Eqs (2.53), (2.54) and (2.55) into 0,0, and 1 and are given as following:

$$\begin{aligned} F_{1u} &= -K_1 x_{1u} \\ F_{2u} &= -K_2 x_{2u} \\ F_{3u} &= TF - K_3 x_{3u} + TTR \end{aligned} \quad (2.70)$$

2.5.4 Current-Vortex Model

In this section, the mathematical model of vortex-induced vibration with internal flow is presented using the two dimensional, coupled wake oscillator model proposed by Iwan and Blevins (1974) for the Reynolds number of 10^3 to 10^5 . There are many vortex excitation model such as harmonic model, wake oscillator model, etc. Of those models, Iwan-Blevin's model has a advantage as a numerical model. This model has a nature of self-excited vortex shedding which means that the fluid behavior may be modeled by a simple, nonlinear, self-excited oscillator. Attention is confined to plane vibrations ($x_2 = 0$) transverse to a steady current in direction x_2 . In-line vibration is treated in the usual way and no coupling is considered in this model (see Figure 2.5).

First, a so-called hidden flow variable w is defined as a fluid motion transverse to a steady current and it, in turn, produces transverse force to cylinder. This transverse force,

which induces the motion of the cylinder is a forcing component in the oscillator equation. The net force per unit length on cylinder is evaluated using the momentum equation for the control volume containing cylinder as shown in Figure 2.5 and is given by

$$q_1 = a_4 \rho_w D_o u_c (\dot{w} - \dot{x}_1) \quad (2.71)$$

and the fluid oscillator equation is

$$\ddot{w} + \omega_s^2 w = (a'_1 - a'_4) u_c \dot{w}/D_o - a'_2 \dot{w}^3/(u_c D_o) + a'_4 u_c \dot{x}_1/D_o \quad (2.72)$$

where ω_s is the vortex shedding frequency given by $\omega_s = 2\pi S u_c/D_o$ and Strouhal number S is taken as equal to 0.20.

On the basis of the results of various vortex experiments for subcritical Reynolds numbers, Iwan and Blevins choose

$$\begin{aligned} a'_1 &= a_1/a_o, \quad a'_2 = a_2/a_o, \quad a'_4 = a_4/a_o \\ a_o &= 0.48, \quad a_1 = 0.44, \quad a_2 = 0.20, \quad a_4 = 0.38 \end{aligned} \quad (2.73)$$

Later, Blevins considered the spanwise coupling along the riser because portions of the riser span are not in resonance for nonuniform flow (1990). In order to apply the model to nonuniform flow, the span of the riser is divided into segments that are within the resonance band and the remainder, which is outside the resonance band.

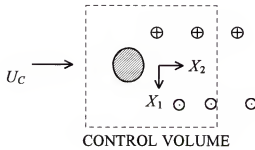


Figure 2.5: Control Volume Containing a Cylinder Shedding Vortices

Define a parameter $p(x_3)$ that defines the spanwise region of the resonance band:

$$p(x_3) = \begin{cases} 1, & \text{if lock-in band, } (\alpha f_s < f_i < \beta f_s) \\ 0, & \text{if no lock-in} \end{cases}$$

where α and β specify the lock-in band and $f_s = 2\pi/\omega_s$ is the stationary cylinder vortex shedding frequency. Typically $\alpha = 0.6$, $\beta = 1.4$ for lightly damped, large amplitude motion. The parameter $p(x_3)$ will step between 0 and 1 along the span, depending on the current profile.

The evaluation of the net force on cylinder under the consideration of the spanwise coupling is obtained from the modification of Eq. (2.71) and is given by

$$q_1 = a_4 \rho_w D_O u_c (\dot{w} - \dot{x}_1) p(x_3) - \frac{1}{2} \rho_w u_c D_o C_D \dot{x}_1 (1 - p(x_3)) \quad (2.74)$$

Since the model derived by Blevin and Iwan is based on the assumption of small amplitude, this model is applied to the approximated equation of motion (2.61), which gives

$$\begin{aligned} m_i \ddot{x}_1 + 2m_f V_1 \dot{x}_1' + m_f V_1^2 x_1'' + (E I x_1'')'' - w x_1' - T_c x_1'' \\ = a_4 \rho_w D_O u_c (\dot{w} - \dot{x}_1) p(x_3) - \frac{1}{2} \rho_w u_c D_o C_D \dot{x}_1 (1 - p(x_3)) \end{aligned} \quad (2.75)$$

From above equation, we can recognize that the riser system with internal flow will respond resonantly to vortex shedding along the part of its span in the lock-in band and the system oscillation will be damped by the external fluid out of that part.

CHAPTER 3 FINITE ELEMENT MODEL OF THE SYSTEM

This chapter formulates the numerical model of governing equations and boundary conditions using finite element method. Variational statement for the boundary value problem is introduced, that is, the weak form of governing equation is derived. Sequentially, the incremental operator is applied on the weak form in order to derive the incremental equilibrium equation and then, Galerkin's decomposition method is performed to construct approximations to the solutions of the problem. Finally incorporation of the usual cubic shape functions yields the matrix dynamic equilibrium equations constructed in terms of the unknown deformations at nodal points. In addition, the finite element formulation for vortex induced vibration is made through the same procedure except applying incremental operator.

3.1 Variational Statement for the problem

The mathematical model of the present problem has been derived in chapter 2 and represented in vector form in Eq. (2.49). To produce a variational statement, we begin by considering Eq. (2.49) as a two point boundary value problem

$$m_i \ddot{\mathbf{r}} + 2m_f V_1 \dot{\mathbf{r}}' + m_f V_1^2 \mathbf{r}'' + (EI \mathbf{r}'')' - [(T_e - EI \kappa^2) \mathbf{r}']' - H(\mathbf{r}' \times \mathbf{r}'')' = \mathbf{q}, \quad 0 < s_1 < L_1 \quad (3.1)$$

where

$$\mathbf{r} = x_1(s_1) \hat{i} + x_2(s_1) \hat{j} + (s + x_3(s_1)) \hat{k}$$

$$L_1 = \int_0^L (1 + \varepsilon_i) ds, \quad (L = \text{Riser length}) \quad (3.2)$$

This is partial nonlinear differential equation, with respect to time and deformed length,

of fourth order with variable coefficients defined in a domain Ω given as the interval $0 < s_1 < L_1$. The variable coefficients are smooth bounded function and the value of EI is always positive. Besides Eq. (2.49), we have nonlinear boundary conditions at top and bottom of riser, i.e., $s_1 = 0$ and $s_1 = L_1$. Equations (2.53) - (2.55) represents the top boundary conditions and the fixed-hinge support are applied at the lower end, that is, those boundary conditions can be given as

$$\begin{aligned}
 \text{at } s_1 = 0, \quad & \mathbf{r} = 0 \quad \text{and} \quad \mathbf{M} = 0 \\
 \text{at } s_1 = L_1, \quad & F_1 = (\text{TF}) \hat{t} \cdot \hat{i} - K_1 x_{1u} \\
 & F_2 = (\text{TF}) \hat{t} \cdot \hat{j} - K_2 x_{2u} \\
 & F_3 = (\text{TF}) \hat{t} \cdot \hat{k} - K_3 x_{3u} + \text{TTR}
 \end{aligned} \tag{3.3}$$

and the torsional moment is given at the lower or upper end.

The weak form of the mathematical model is constructed as follows: find the vector function $\mathbf{r}$ such that the differential equation, together with the boundary conditions, are satisfied in the sense of weighted average. To express the weighted averages of differential equation, we first construct the residual error function vector $\mathbf{R}$ of Eq. (3.1) such as

$$\begin{aligned}
 \mathbf{R}(s_1) = & m_i \ddot{\mathbf{r}} + 2m_f V_1 \dot{\mathbf{r}}' + m_f V_1^2 \mathbf{r}'' + (\text{EI} \mathbf{r}'')'' \\
 & - [(T_e - \text{EI} \kappa^2) \mathbf{r}']' - \mathbf{H}(\mathbf{r}' \times \mathbf{r}'')' - \mathbf{q}, \quad s_1 \in \Omega
 \end{aligned} \tag{3.4}$$

Multiply $\mathbf{R}$ by a sufficiently smooth test function vector, $\bar{\mathbf{r}} = \bar{x}_1 \hat{i} + \bar{x}_2 \hat{j} + \bar{x}_3 \hat{k}$, defined over the entire domain, and integrate with respect to the deformed arclength.

$$\int_{\Omega} \mathbf{R} \cdot \bar{\mathbf{r}} ds_1 \tag{3.5}$$

The weak form of the model is constructed by letting the above integral equal to zero.

The finite difference form of the model is given by

$$\sum_{i=1}^N \int_{\Omega_i} \mathbf{R} \cdot \bar{\mathbf{r}} ds_1 = \sum_{i=1}^N \int_0^{l_i} \mathbf{R} \cdot \bar{\mathbf{r}} ds_1 = 0 \quad (3.6)$$

where $l_{1i} = \int_0^{l_i} (1 + \varepsilon_i) ds$, l_i = the length of i-th element

Substitute Eq. (3.4) into (3.6) and separate the integrand for each subdomain to give

$$\begin{aligned} & \int_0^{l_i} [m_i \ddot{\mathbf{r}} + 2m_f V_1 \dot{\mathbf{r}}' + m_f V_1^2 \mathbf{r}'' + (\text{EI} \mathbf{r}'')'] \\ & - \{(\mathbf{T}_e - \text{EI} \kappa^2) \mathbf{r}'\}' - \mathbf{H}(\mathbf{r}' \times \mathbf{r}'')' - \mathbf{q} \} \cdot \bar{\mathbf{r}} ds_1 = 0 \end{aligned} \quad (3.7)$$

Integrate by parts to reduce the order of differentiation to introduce

$$\begin{aligned} & \int_0^{l_i} (\text{EI} \mathbf{r}'')' \cdot \bar{\mathbf{r}} ds_1 = (\text{EI} \mathbf{r}'')' \cdot \bar{\mathbf{r}} \Big|_0^{l_{1i}} - (\text{EI} \mathbf{r}'') \cdot \bar{\mathbf{r}}' \Big|_0^{l_{1i}} + \int_0^{l_i} (\text{EI} \mathbf{r}'') \cdot \bar{\mathbf{r}}'' ds_1 \\ & \int_0^{l_i} m_f V_1^2 \mathbf{r}'' \cdot \bar{\mathbf{r}} ds_1 = m_f V_1^2 \mathbf{r}' \cdot \bar{\mathbf{r}} \Big|_0^{l_{1i}} - \int_0^{l_i} m_f V_1^2 \mathbf{r}' \cdot \bar{\mathbf{r}}' ds_1 \\ & \int_0^{l_i} \{(\mathbf{T}_e - \text{EI} \kappa^2) \mathbf{r}'\}' \cdot \bar{\mathbf{r}} ds_1 = \{(\mathbf{T}_e - \text{EI} \kappa^2) \mathbf{r}'\} \cdot \bar{\mathbf{r}} \Big|_0^{l_{1i}} - \int_0^{l_i} \{(\mathbf{T}_e - \text{EI} \kappa^2) \mathbf{r}'\} \cdot \bar{\mathbf{r}}' ds_1 \\ & \int_0^{l_i} \mathbf{H}(\mathbf{r}' \times \mathbf{r}'')' \cdot \bar{\mathbf{r}} ds_1 = \mathbf{H}(\mathbf{r}' \times \mathbf{r}'') \cdot \bar{\mathbf{r}} \Big|_0^{l_{1i}} - \int_0^{l_i} \mathbf{H}(\mathbf{r}' \times \mathbf{r}'') \cdot \bar{\mathbf{r}}' ds_1 \end{aligned} \quad (3.8)$$

Combining Eq. (3.8) into (3.7), we get

$$\begin{aligned} & \int_0^{l_i} m_i \ddot{\mathbf{r}} \cdot \bar{\mathbf{r}} ds_1 + \int_0^{l_i} 2m_f V_1 \dot{\mathbf{r}}' \cdot \bar{\mathbf{r}} ds_1 - \int_0^{l_i} m_f V_1^2 \mathbf{r}' \cdot \bar{\mathbf{r}}' ds_1 + \int_0^{l_i} (\text{EI} \mathbf{r}'') \cdot \bar{\mathbf{r}}'' ds_1 \\ & + \int_0^{l_i} \{(\mathbf{T}_e - \text{EI} \kappa^2) \mathbf{r}'\} \cdot \bar{\mathbf{r}}' ds_1 + \int_0^{l_i} \mathbf{H}(\mathbf{r}' \times \mathbf{r}'') \cdot \bar{\mathbf{r}}' ds_1 = -m_f V_1^2 \mathbf{r}' \cdot \bar{\mathbf{r}} \Big|_0^{l_{1i}} + (\text{EI} \mathbf{r}'') \cdot \bar{\mathbf{r}}' \Big|_0^{l_{1i}} \\ & + [-(\text{EI} \mathbf{r}'')' + \{(\mathbf{T}_e - \text{EI} \kappa^2) \mathbf{r}'\} + \mathbf{H}(\mathbf{r}' \times \mathbf{r}'') \cdot \bar{\mathbf{r}}] \Big|_0^{l_{1i}} + \int_0^{l_i} \mathbf{q} \cdot \bar{\mathbf{r}} ds_1 \end{aligned} \quad (3.9)$$

From Eq. (2.14) and the fact stated in the first paragraph of Section 2.5.1, we have

$$\mathbf{F} = -(\mathbf{E}\mathbf{I}\mathbf{r}'')' + \{(\mathbf{T}_e - \mathbf{E}\mathbf{I}\kappa^2)\mathbf{r}'\} + \mathbf{H}(\mathbf{r}' \times \mathbf{r}'') \quad (3.10)$$

and also we have the following relationship

$$\hat{\mathbf{t}} \times \mathbf{M} = \hat{\mathbf{t}} \times (\mathbf{E}\mathbf{I}\kappa\hat{\mathbf{b}} + \mathbf{H}\hat{\mathbf{t}}) = -\mathbf{E}\mathbf{I}\kappa\hat{\mathbf{b}} = -\mathbf{E}\mathbf{I}\mathbf{r}'' \quad (3.11)$$

Rewrite Eq. (3.9) using (3.10) and (3.11) to give

$$\begin{aligned} & \int_0^{l_1} m_t \bar{\mathbf{r}} \cdot \bar{\mathbf{r}} \, ds_1 + \int_0^{l_1} 2m_f V_1 \dot{\mathbf{r}}' \cdot \bar{\mathbf{r}} \, ds_1 - \int_0^{l_1} m_f V_1^2 \mathbf{r}' \cdot \bar{\mathbf{r}}' \, ds_1 + \int_0^{l_1} (\mathbf{E}\mathbf{I}\mathbf{r}'') \cdot \bar{\mathbf{r}}'' \, ds_1 \\ & + \int_0^l \{(\mathbf{T}_e - \mathbf{E}\mathbf{I}\kappa^2)\mathbf{r}'\} \cdot \bar{\mathbf{r}}' \, ds_1 + \int_0^l \mathbf{H}(\mathbf{r}' \times \mathbf{r}'') \cdot \bar{\mathbf{r}}' \, ds_1 = \{-m_f V_1^2 \mathbf{r}' + \mathbf{F}\} \cdot \bar{\mathbf{r}} \Big|_0^{l_1} \\ & - (\hat{\mathbf{t}} \times \mathbf{M}) \cdot \bar{\mathbf{r}}' \Big|_0^{l_1} + \int_0^{l_1} \mathbf{q} \cdot \bar{\mathbf{r}} \, ds_1 \end{aligned} \quad (3.12)$$

Equation (3.12) represents the weak formulation of the boundary value problem for each subdomain that transforms the differential equation over the subdomain and boundary condition at the both end into a single equation in which all of the features of the solution and the discontinuous data are intrinsically present. Moreover, from the substitution of Eq. (3.12) into (3.6), we can define the same variational statement over the entire system. The difference is that the boundary conditions represent ones at the both end of whole domain due to the cancellation of the boundary conditions of each subdomain.

3.2 Incremental Weak Formulation

In previous section, the weak form of governing equation has been derived, i.e., multiplying both sides of equation by a virtual displacement or a test function and integrating over the length of a finite element. And then, finite approximations on the weak form yields the matrix equilibrium equation. This is a usual procedure of the finite element approximation. However, the application of this procedure may show inaccuracy in

solution or inefficiency in convergence resulted from so many iteration for the nonlinear terms in the equation. And therefore, the application of incremental operator on the weak form of governing equation (Oden, J.T., 1972) is required for the successful iteration and the incremental matrix equilibrium equation can be obtained from the incremental weak form.

The incremental operator Δ has the meaning of a difference operator, i.e.,

$$\Delta x = x_{k+1} - x_k$$

where x is an independent variable and the subscript indicates the increment number.

The operator can be applied to a function f such as:

$$\Delta f = f_{k+1} - f_k \quad (3.13)$$

and equation (3.13) can be rewritten by using Taylor's expansion. For example, if f is a function of independent variables x and y , Δf can be expressed in

$$\Delta f \equiv \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y + O(\Delta x^2, \Delta y^2, \Delta x \Delta y) \quad (3.14)$$

Now, applying incremental operator Δ on Eq. (3.12) with independent variable $\mathbf{r}$ gives

$$\begin{aligned} & \int_0^{l_1} m_t \Delta \ddot{\mathbf{r}} \cdot \bar{\mathbf{r}} \, ds_1 + \int_0^{l_1} 2m_f V_1 \Delta \mathbf{r}' \cdot \bar{\mathbf{r}} \, ds_1 - \int_0^{l_1} m_f V_1^2 \Delta \mathbf{r}' \cdot \bar{\mathbf{r}}' \, ds_1 \\ & + \int_0^{l_1} (EI \Delta \mathbf{r}'') \cdot \bar{\mathbf{r}}'' \, ds_1 + \int_0^{l_1} \{ \Delta(T_e - EI \kappa^2) \mathbf{r}' + (T_e - EI \kappa^2) \Delta \mathbf{r}' \} \cdot \bar{\mathbf{r}}' \, ds_1 \\ & + \int_0^{l_1} H(\Delta \mathbf{r}' \times \mathbf{r}'' + \mathbf{r}' \times \Delta \mathbf{r}'') \cdot \bar{\mathbf{r}}' \, ds_1 = \{ -m_f V_1^2 \Delta \mathbf{r}' + \Delta \mathbf{F} \} \cdot \bar{\mathbf{r}} \Big|_0^{l_1} \\ & + EI \Delta \mathbf{r}'' \cdot \bar{\mathbf{r}}' \Big|_0^{l_1} + \int_0^{l_1} \Delta \mathbf{q} \cdot \bar{\mathbf{r}} \, ds_1 \end{aligned} \quad (3.15)$$

in which

$$\Delta(T_e - EI\kappa^2) = \Delta T_e - EI\Delta(\kappa^2) \quad (3.16)$$

Combine Eqs. (2.1), (2.2), and (2.15) into (3.16), we obtain

$$\Delta T_e - EI\Delta(\kappa^2) = \Delta(\mathbf{F} \cdot \mathbf{r}') - EI\Delta(\mathbf{r}'' \cdot \mathbf{r}'') = \Delta\mathbf{F} \cdot \mathbf{r}' + \mathbf{F} \cdot \Delta\mathbf{r}' - 2EI\Delta\mathbf{r}'' \cdot \mathbf{r}'' \quad (3.17)$$

Substitute the result of (3.17) into (3.15), we finally get

$$\begin{aligned} & \int_0^{l_1} m_t \Delta \bar{\mathbf{r}} \cdot \bar{\mathbf{r}} \, ds_1 + \int_0^{l_1} 2m_f V_1 \Delta \bar{\mathbf{r}}' \cdot \bar{\mathbf{r}} \, ds_1 - \int_0^{l_1} m_f V_1^2 \Delta \bar{\mathbf{r}}' \cdot \bar{\mathbf{r}}' \, ds_1 \\ & + \int_0^{l_1} (EI \Delta \mathbf{r}'') \cdot \bar{\mathbf{r}}'' \, ds_1 + \int_0^{l_1} (T_e - EI\kappa^2) \Delta \mathbf{r}' \cdot \bar{\mathbf{r}}' \, ds_1 + \int_0^{l_1} (\mathbf{F} \cdot \Delta \mathbf{r}') (\mathbf{r}' \cdot \bar{\mathbf{r}}') \, ds_1 \\ & - \int_0^{l_1} (2EI \mathbf{r}'' \cdot \Delta \mathbf{r}'') (\mathbf{r}' \cdot \bar{\mathbf{r}}') \, ds_1 + \int_0^{l_1} H(\Delta \mathbf{r}' \times \mathbf{r}'') \cdot \bar{\mathbf{r}}' \, ds_1 \\ & + \int_0^{l_1} H(\mathbf{r}' \times \Delta \mathbf{r}'') \cdot \bar{\mathbf{r}}' \, ds_1 = \{-m_f V_1^2 \Delta \mathbf{r}' + \Delta \mathbf{F}\} \cdot \bar{\mathbf{r}} \Big|_0^{l_1} + EI \Delta \mathbf{r}'' \cdot \bar{\mathbf{r}}' \Big|_0^{l_1} \\ & + \int_0^{l_1} \Delta \mathbf{q} \cdot \bar{\mathbf{r}} \, ds_1 - \int_0^{l_1} (\Delta \mathbf{F} \cdot \mathbf{r}') (\mathbf{r}' \cdot \bar{\mathbf{r}}') \, ds_1 \end{aligned} \quad (3.18)$$

This is the incremental weak form of the governing equation and the quantities in front of integrals are considered constants for each element so that those terms may be factored out of the integrals. In order to represent Eq. (3.18) into components, the differentiating terms of the position vector or of the virtual displacement vector with respect to time and deformed arc length should be considered and are given by

$$\mathbf{r} = x_1 \hat{i} + x_2 \hat{j} + (x_3 + s) \hat{k}$$

$$\bar{\mathbf{r}} = \bar{x}_1 \hat{i} + \bar{x}_2 \hat{j} + \bar{x}_3 \hat{k}$$

$$\bar{\mathbf{r}}' = \bar{x}'_1 \hat{i} + \bar{x}'_2 \hat{j} + \bar{x}'_3 \hat{k}$$

$$\begin{aligned}
\bar{\mathbf{r}}'' &= \bar{x}_1'' \hat{i} + \bar{x}_2'' \hat{j} + \bar{x}_3'' \hat{k} \\
\mathbf{r}' &= x_1' \hat{i} + x_2' \hat{j} + d_1 \hat{k} \\
\dot{\mathbf{r}}' &= \dot{x}_1' \hat{i} + \dot{x}_2' \hat{j} + d_2 \hat{k} \\
\mathbf{r}'' &= x_1'' \hat{i} + x_2'' \hat{j} + d_3 \hat{k} \\
\kappa^2 &= x_1''^2 + x_2''^2 + d_4 \\
\Delta \mathbf{r}' &= \Delta x_1' \hat{i} + \Delta x_2' \hat{j} + d_5 \hat{k} \\
\Delta \dot{\mathbf{r}}' &= \Delta \dot{x}_1' \hat{i} + \Delta \dot{x}_2' \hat{j} + d_6 \hat{k} \\
\Delta \mathbf{r}'' &= \Delta x_1'' \hat{i} + \Delta x_2'' \hat{j} + d_7 \hat{k} \\
\Delta \ddot{\mathbf{r}} &= \Delta \ddot{x}_1 \hat{i} + \Delta \ddot{x}_2 \hat{j} + \Delta \ddot{x}_3 \hat{k}
\end{aligned} \tag{3.19}$$

where

$$\begin{aligned}
d_1 &= c_1^{1/2} \\
d_2 &= -c_1^{-1/2} c_2 \\
d_3 &= -c_1^{-1/2} c_3 \\
d_4 &= c_1^{-1} c_3^2 \\
d_5 &= -c_1^{-1/2} c_4 \\
d_6 &= -(c_1^{-3/2} c_2 c_4 + c_1^{-1/2} c_5) \\
d_7 &= -(c_1^{-3/2} c_3 c_4 + c_1^{-1/2} c_6) \\
c_1 &= 1 - x_1'^2 - x_2'^2 = (x_3' + 1/(1 + \varepsilon_t))^2 \\
c_2 &= x_1' \dot{x}_1' + x_2' \dot{x}_2' \\
c_3 &= x_1' x_1'' + x_2' x_2'' \\
c_4 &= x_1' \Delta x_1' + x_2' \Delta x_2' \\
c_5 &= \Delta x_1' \dot{x}_1' + x_1' \Delta \dot{x}_1' + \Delta x_2' \dot{x}_2' + x_2' \Delta \dot{x}_2' \\
c_6 &= \Delta x_1' x_1'' + x_1' \Delta x_1'' + \Delta x_2' x_2'' + x_2' \Delta x_2''
\end{aligned} \tag{3.20}$$

3.3 Finite Element Approximations

The construction of a finite element approximation of the problem, such as given in (3.18), is based on Galerkin's decomposition method. Here, we replace (3.18) by a finite dimensional problem of the following form to find the approximate solution vector $\mathbf{r}_h$ such that

$$\begin{aligned}
 & \int_0^{l_1} m_f \Delta \bar{\mathbf{r}}_h \cdot \bar{\mathbf{r}}_h ds_1 + \int_0^{l_1} 2m_f V_1 \Delta \bar{\mathbf{r}}'_h \cdot \bar{\mathbf{r}}_h ds_1 - \int_0^{l_1} m_f V_1^2 \Delta \bar{\mathbf{r}}'_h \cdot \bar{\mathbf{r}}'_h ds_1 \\
 & + \int_0^{l_1} (EI \Delta \bar{\mathbf{r}}''_h) \cdot \bar{\mathbf{r}}''_h ds_1 + \int_0^{l_1} (\Gamma_e - EI \kappa^2) \Delta \bar{\mathbf{r}}'_h \cdot \bar{\mathbf{r}}'_h ds_1 + \int_0^{l_1} (\mathbf{F} \cdot \Delta \bar{\mathbf{r}}'_h) (\bar{\mathbf{r}}'_h \cdot \bar{\mathbf{r}}'_h) ds_1 \\
 & - \int_0^{l_1} (2EI \bar{\mathbf{r}}''_h \cdot \Delta \bar{\mathbf{r}}''_h) (\bar{\mathbf{r}}'_h \cdot \bar{\mathbf{r}}'_h) ds_1 + \int_0^{l_1} H (\Delta \bar{\mathbf{r}}'_h \times \bar{\mathbf{r}}''_h) \cdot \bar{\mathbf{r}}'_h ds_1 \\
 & + \int_0^{l_1} H (\bar{\mathbf{r}}'_h \times \Delta \bar{\mathbf{r}}''_h) \cdot \bar{\mathbf{r}}'_h ds_1 = \{-m_f V_1^2 \Delta \bar{\mathbf{r}}'_h + \Delta \mathbf{F}\} \cdot \bar{\mathbf{r}}_h \Big|_0^{l_1} + EI \Delta \bar{\mathbf{r}}''_h \cdot \bar{\mathbf{r}}'_h \Big|_0^{l_1} \\
 & + \int_0^{l_1} \Delta \mathbf{q} \cdot \bar{\mathbf{r}}_h ds_1 - \int_0^{l_1} (\Delta \mathbf{F} \cdot \bar{\mathbf{r}}'_h) (\bar{\mathbf{r}}'_h \cdot \bar{\mathbf{r}}'_h) ds_1 \quad (3.21)
 \end{aligned}$$

The nodal values of the approximate solution are unknown functions of time and consist of not only the deflections at the nodes, but also the rotations. In other words, after constructing interpolation functions on a suitable mesh, we take the approximate solution to be of the form

$$\mathbf{r}_h = x_{1h} \hat{i} + x_{2h} \hat{j} + (x_{3h} + s) \hat{k}$$

$$\text{and} \quad x_{jh}(s_1, t) = \sum_{i=1}^4 x_{ji}(t) N_i(s_1) \quad j = 1, 2, 3 \quad (3.22)$$

where x_{j1}, x_{j3} and x_{j2}, x_{j4} represent the deflections and the rotations at node, and N_i is the basis or interpolation function.

As an interpolation function, Hermite polynomials are constructed by matching together element shape functions in the usual way. Let $\tilde{\Omega}$ denote a master element, with coordinate ξ , $-1 \leq \xi \leq 1$. Note that the specification of x_{jh} and x'_{jh} at the endpoints of an element involves four conditions. Since a complete cubic in ξ contains exactly four parameters, we can construct cubic shape functions $\tilde{N}_i$ which exhibit the necessary properties:

$$\begin{aligned}\tilde{N}_1(-1) &= 1, \quad \tilde{N}_1(1) = 0, \quad d\tilde{N}_1(-1)/d\xi = d\tilde{N}_1(1)/d\xi = 0 \\ \tilde{N}_3(-1) &= 0, \quad \tilde{N}_3(1) = 1, \quad d\tilde{N}_3(-1)/d\xi = d\tilde{N}_3(1)/d\xi = 0 \\ \tilde{N}_2(-1) &= \tilde{N}_2(1) = 0, \quad d\tilde{N}_2(-1)/d\xi = 1, \quad d\tilde{N}_2(1)/d\xi = 0 \\ \tilde{N}_4(-1) &= \tilde{N}_4(1) = 0, \quad d\tilde{N}_4(-1)/d\xi = 0, \quad d\tilde{N}_4(1)/d\xi = 1\end{aligned}\quad (3.23)$$

These conditions define a set of unique local cubic Hermite polynomial shape functions,

$$\begin{aligned}\tilde{N}_1(\xi) &= \frac{1}{4}(2 - 3\xi + \xi^3), \quad \tilde{N}_3(\xi) = \frac{1}{4}(2 + 3\xi - \xi^3) \\ \tilde{N}_2(\xi) &= \frac{1}{4}(1 - \xi - \xi^2 + \xi^3), \quad \tilde{N}_4(\xi) = \frac{1}{4}(-1 - \xi + \xi^2 + \xi^3)\end{aligned}\quad (3.24)$$

which are illustrated in Fig. 3.1.

The restriction x_{jh}^i of a approximate solution x_{jh} to an element Ω_i between nodes i and $i+1$ can be written as

$$x_{jh}^i(s_1, t) = \tilde{x}_{jh}(\xi(s_1), t) \quad (3.25)$$

where

$$\xi(s_1) = \frac{2s_1 - (s_{1i} + s_{i+1})}{s_{1i+1} - s_{1i}} \quad (3.26)$$

and x_h is the approximated solution referred to the master element,

$$x_h(\xi, t) = \sum_{j=1}^4 x_{ji}(t) \tilde{N}_i(\xi) \quad (3.27)$$

The element shape function N_i is obtained from (3.24) and (3.26) as

$$N_i(s_1) = \tilde{N}_i(\xi(s_1)) \quad (3.28)$$

Combining corresponding shape functions N_i at node i determines the global basis function N_i^j at node i (Fig. 3.1).

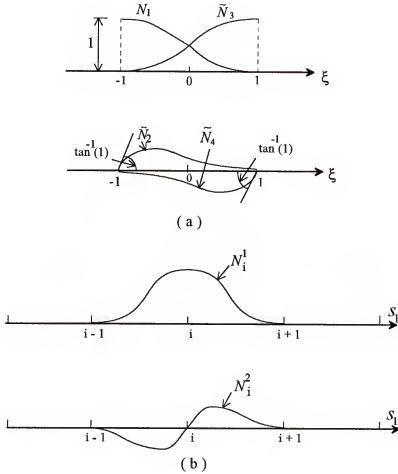


Figure 3.1: (a) Hermite Cubic Shape Functions on a Master Element and (b) Corresponding Global Basis Functions at a Node i in the Mesh.

Having selected the basis functions, the incremental form of a approximate solution and the test function can be written similarly

$$\begin{aligned}\Delta x_{jh}(s_1, t) &= \sum_{j=1}^4 \Delta x_{ji}(t) N_i(s_1) \\ \bar{x}_{jh}(s_1, t) &= \sum_{j=1}^4 \bar{x}_{ji}(t) N_i(s_1)\end{aligned}\quad (3.29)$$

The substitution of (3.29) into (3.21) with the consideration of (3.19) introduces the series form of the incremental weak form. Manipulating the series form and writing term by term,

$$\begin{aligned}\int_0^{t_1} m_t \Delta \dot{\mathbf{r}}_h \cdot \bar{\mathbf{r}}_h ds_1 &= \int_0^{t_1} m_t (\Delta \ddot{x}_{1h} \bar{x}_{1h} + \Delta \ddot{x}_{2h} \bar{x}_{2h} + \Delta \ddot{x}_{3h} \bar{x}_{3h}) ds_1 \\ &= \int_0^{t_1} m_t \left\{ \sum_{j=1}^3 \left(\sum_{i=1}^4 \Delta \ddot{x}_{ji} N_i \right) \left(\sum_{i=1}^4 \Delta \bar{x}_{ji} N_i \right) \right\} ds_1 \\ &= \sum_{k=1}^3 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{t_1} m_t N_i N_j ds_1 \right) \Delta \ddot{x}_{kj} \right\} \bar{x}_{ki} \right]\end{aligned}\quad (3.30)$$

$$\begin{aligned}\int_0^{t_1} 2m_f V_1 \Delta \dot{\mathbf{r}}'_h \cdot \bar{\mathbf{r}}_h ds_1 &= \int_0^{t_1} 2m_f V_1 (\Delta \dot{x}'_{1h} \bar{x}_{1h} + \Delta \dot{x}'_{2h} \bar{x}_{2h} + d_6 \bar{x}_{3h}) ds_1 \\ &= \int_0^{t_1} 2m_f V_1 \left\{ \sum_{j=1}^2 \left(\sum_{i=1}^4 \Delta \dot{x}_{ji} N'_i \right) \left(\sum_{i=1}^4 \bar{x}_{ji} N_i \right) + d_6 \sum_{i=1}^4 \bar{x}_{3i} N_i \right\} ds_1 \\ &= \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{t_1} 2m_f V_1 N_i N'_j ds_1 \right) \Delta \dot{x}_{kj} \right\} \bar{x}_{ki} \right] + \sum_{i=1}^4 \left(\int_0^{t_1} 2m_f V_1 d_6 N_i ds_1 \right) \bar{x}_{3i}\end{aligned}\quad (3.31)$$

$$\begin{aligned}\int_0^{t_1} -m_f V_1^2 \Delta \mathbf{r}'_h \cdot \bar{\mathbf{r}}_h ds_1 &= \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{t_1} -m_f V_1^2 N'_i N'_j ds_1 \right) \Delta x_{kj} \right\} \bar{x}_{ki} \right] \\ &+ \sum_{i=1}^4 \left(\int_0^{t_1} -m_f V_1^2 d_5 N'_i ds_1 \right) \bar{x}_{3i}\end{aligned}\quad (3.32)$$

$$\begin{aligned}
\int_0^{l_1} \text{EI} \Delta \mathbf{r}_h'' \cdot \bar{\mathbf{r}}_h'' ds_1 &= \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{l_1} \text{EI} N_i'' N_j'' ds_1 \right) \Delta x_{kj} \right\} \bar{x}_{ki} \right] \\
&+ \sum_{i=1}^4 \left(\int_0^{l_1} \text{EI} d_7 N_i'' ds_1 \right) \bar{x}_{3i}
\end{aligned} \quad (3.33)$$

$$\begin{aligned}
\int_0^{l_1} (\text{T}_e - \text{EI} \kappa^2) \Delta \mathbf{r}_h' \cdot \bar{\mathbf{r}}_h' ds_1 &= \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{l_1} (\text{T}_e - \text{EI} \kappa^2) N_i' N_j' ds_1 \right) \Delta x_{kj} \right\} \bar{x}_{ki} \right] \\
&+ \sum_{i=1}^4 \left(\int_0^{l_1} (\text{T}_e - \text{EI} \kappa^2) d_5 N_i' ds_1 \right) \bar{x}_{3i}
\end{aligned} \quad (3.34)$$

$$\begin{aligned}
&\int_0^{l_1} (\mathbf{F} \cdot \Delta \mathbf{r}_h') (\mathbf{r}_h' \cdot \bar{\mathbf{r}}_h') ds_1 \\
&= \int_0^{l_1} (F_1 \Delta x'_{1h} + F_2 \Delta x'_{2h} + F_3 d_5) (x'_{1h} \bar{x}'_{1h} + x'_{2h} \bar{x}'_{2h} + d_1 \bar{x}'_{3h}) ds \\
&= \sum_{i=1}^4 \sum_{j=1}^4 \left\{ \left(\int_0^{l_1} F_1 x'_{1h} N_i' N_j' ds_1 \right) \Delta x_{1j} + \left(\int_0^{l_1} F_2 x'_{1h} N_i' N_j' ds_1 \right) \Delta x_{2j} \right\} \bar{x}_{1i} \\
&+ \sum_{i=1}^4 \sum_{j=1}^4 \left\{ \left(\int_0^{l_1} F_1 x'_{2h} N_i' N_j' ds_1 \right) \Delta x_{1j} + \left(\int_0^{l_1} F_2 x'_{2h} N_i' N_j' ds_1 \right) \Delta x_{2j} \right\} \bar{x}_{2i} \\
&+ \sum_{i=1}^4 \sum_{j=1}^4 \left\{ \left(\int_0^{l_1} F_1 d_1 N_i' N_j' ds_1 \right) \Delta x_{1j} + \left(\int_0^{l_1} F_2 d_1 N_i' N_j' ds_1 \right) \Delta x_{2j} \right\} \bar{x}_{3i} \\
&+ \sum_{i=1}^4 \left(\int_0^{l_1} F_3 d_5 x'_{1h} N_i' ds_1 \right) \bar{x}_{1i} + \sum_{i=1}^4 \left(\int_0^{l_1} F_3 d_5 x'_{2h} N_i' ds_1 \right) \bar{x}_{2i} \\
&+ \sum_{i=1}^4 \left(\int_0^{l_1} F_3 d_5 d_1 N_i' ds_1 \right) \bar{x}_{3i}
\end{aligned} \quad (3.35)$$

$$\begin{aligned}
&\int_0^{l_1} -2 \text{EI} (\mathbf{r}_h'' \cdot \Delta \mathbf{r}_h'') (\mathbf{r}_h' \cdot \bar{\mathbf{r}}_h') ds_1 \\
&= \int_0^{l_1} -2 \text{EI} (x''_{1h} \Delta x''_{1h} + x''_{2h} \Delta x''_{2h} + d_3 d_7) (x'_{1h} \bar{x}'_{1h} + x'_{2h} \bar{x}'_{2h} + d_1 \bar{x}'_{3h}) ds_1
\end{aligned}$$

$$\begin{aligned}
&= \sum_{i=1}^4 \sum_{j=1}^4 \{ (\int_0^{l_1} -2EI x'_{1h} x''_{1h} N'_i N''_j ds_1) \Delta x_{1j} + (\int_0^{l_1} -2EI x'_{1h} x''_{2h} N'_i N''_j ds_1) \Delta x_{2j} \} \bar{x}_{1i} \\
&+ \sum_{i=1}^4 \sum_{j=1}^4 \{ (\int_0^{l_1} -2EI x'_{2h} x''_{1h} N'_i N''_j ds_1) \Delta x_{1j} + (\int_0^{l_1} -2EI x'_{2h} x''_{2h} N'_i N''_j ds_1) \Delta x_{2j} \} \bar{x}_{2i} \\
&+ \sum_{i=1}^4 \sum_{j=1}^4 \{ (\int_0^{l_1} -2EI x''_{1h} d_1 N'_i N''_j ds_1) \Delta x_{1j} + (\int_0^{l_1} -2EI x''_{2h} d_1 N'_i N''_j ds_1) \Delta x_{2j} \} \bar{x}_{3i} \\
&+ \sum_{i=1}^4 (\int_0^{l_1} -2EI d_3 d_7 x'_{1h} N'_i ds_1) \bar{x}_{1i} + \sum_{i=1}^4 (\int_0^{l_1} -2EI d_3 d_7 x'_{2h} N'_i ds_1) \bar{x}_{2i} \\
&+ \sum_{i=1}^4 (\int_0^{l_1} -2EI d_1 d_3 d_7 N'_i ds_1) \bar{x}_{3i} \quad (3.36)
\end{aligned}$$

$$\begin{aligned}
&\int_0^{l_1} H(\Delta \mathbf{r}'_h \times \mathbf{r}''_h) \cdot \bar{\mathbf{r}}'_h ds_1 \\
&= \int_0^{l_1} H \{ (d_3 \Delta x'_{2h} - d_5 x''_{2h}) \bar{x}'_{1h} + (d_5 x''_{1h} - d_3 \Delta x'_{1h}) \bar{x}'_{2h} + (x''_{2h} \Delta x'_{1h} - x'_{1h} \Delta x'_{2h}) \bar{x}'_{3h} \} ds_1 \\
&= \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} H d_3 N'_i N''_j ds_1) \Delta x_{2j} \bar{x}_{1i} + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} -H d_3 N'_i N''_j ds_1) \Delta x_{1j} \bar{x}_{2i} \\
&+ \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} H x''_{2h} N'_i N''_j ds_1) \Delta x_{1j} \bar{x}_{3i} + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} -H x''_{1h} N'_i N''_j ds_1) \Delta x_{2j} \bar{x}_{3i} \\
&+ \sum_{i=1}^4 (\int_0^{l_1} -H d_5 x''_{2h} N'_i ds_1) \bar{x}_{1i} + \sum_{i=1}^4 (\int_0^{l_1} H d_5 x''_{1h} N'_i ds_1) \bar{x}_{2i} \quad (3.37)
\end{aligned}$$

$$\begin{aligned}
&\int_0^{l_1} H(\mathbf{r}'_h \times \Delta \mathbf{r}''_h) \cdot \bar{\mathbf{r}}'_h ds_1 \\
&= \int_0^{l_1} H \{ (d_7 x'_{2h} - d_1 \Delta x''_{2h}) \bar{x}'_{1h} + (d_1 \Delta x''_{1h} - d_7 x'_{1h}) \bar{x}'_{2h} + (x'_{1h} \Delta x''_{2h} - x'_{2h} \Delta x''_{1h}) \bar{x}'_{3h} \} ds_1 \\
&= \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} -H d_1 N'_i N''_j ds_1) \Delta x_{2j} \bar{x}_{1i} + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} H d_1 N'_i N''_j ds_1) \Delta x_{1j} \bar{x}_{2i} \\
&+ \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} H x'_{1h} N'_i N''_j ds_1) \Delta x_{2j} \bar{x}_{3i} + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} -H x'_{2h} N'_i N''_j ds_1) \Delta x_{1j} \bar{x}_{3i}
\end{aligned}$$

$$+ \sum_{i=1}^4 \left(\int_0^{l_1} H d_7 x'_{2h} N'_i ds_1 \right) \bar{x}_{1i} + \sum_{i=1}^4 \left(\int_0^{l_1} -H d_7 x'_{1h} N'_i ds_1 \right) \bar{x}_{2i} \quad (3.38)$$

$$\begin{aligned} \int_0^{l_1} \Delta \mathbf{q} \cdot \bar{\mathbf{r}}_h ds_1 &= \int_0^{l_1} (\Delta q_1 \bar{x}_{1h} + \Delta q_2 \bar{x}_{2h} + \Delta q_3 \bar{x}_{3h}) ds_1 \\ &= \sum_{k=1}^3 \sum_{i=1}^4 \left(\int_0^{l_1} \Delta q_k N_i ds_1 \right) x_{ki} \end{aligned} \quad (3.39)$$

$$\begin{aligned} &\int_0^{l_1} -(\Delta \mathbf{F} \cdot \mathbf{r}'_h)(\mathbf{r}'_h \cdot \bar{\mathbf{r}}'_h) ds_1 \\ &= \int_0^{l_1} -(\Delta F_1 x'_{1h} + \Delta F_2 x'_{2h} + \Delta F_3 d_1)(x'_{1h} \bar{x}'_{1h} + x'_{2h} \bar{x}'_{2h} + d_1 \bar{x}'_{3h}) ds \\ &= \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \int_0^{l_1} -(\Delta F_1 x'_{1h} + \Delta F_2 x'_{2h} + \Delta F_3 d_1) x'_{kh} N'_i ds_1 \right\} \bar{x}_{ki} \right] \\ &\quad + \sum_{i=1}^4 \left\{ \int_0^{l_1} -(\Delta F_1 x'_{1h} + \Delta F_2 x'_{2h} + \Delta F_3 d_1) d_1 N'_i ds_1 \right\} \bar{x}'_{3i} \end{aligned} \quad (3.40)$$

$$\begin{aligned} (-m_i V_1^2 \Delta \mathbf{r}'_h + \Delta \mathbf{F}) \cdot \bar{\mathbf{r}}_h \Big|_0^{l_{1i}} &= -m_i V_1^2 \Delta \mathbf{r}'_h \cdot \bar{\mathbf{r}}_h + \Delta \mathbf{F} \cdot \bar{\mathbf{r}}_h \Big|_0^{l_{1i}} \\ &= -m_i V_1^2 (\Delta x'_{1h} \bar{x}_{1h} + \Delta x'_{2h} \bar{x}_{2h} + d_5 \bar{x}_{3h}) + (\Delta F_1 \bar{x}_{1h} + \Delta F_2 \bar{x}_{2h} + \Delta F_3 \bar{x}_{3h}) \Big|_0^{l_{1i}} \\ &= [-m_i V_1^2 \{ \sum_{k=1}^2 \left(\sum_{i=1}^4 \sum_{j=1}^4 N'_i N'_j \Delta x_{ki} \bar{x}_{kj} \right) + d_5 \left(\sum_{i=1}^4 N_i \bar{x}_{3i} \right) \} + \sum_{k=1}^3 \{ \Delta F_k \left(\sum_{i=1}^4 N_i \bar{x}_{ki} \right) \}] \Big|_0^{l_{1i}} \\ &= -m_i V_1^2 \{ (\Delta x_{14} \bar{x}_{13} - \Delta x_{12} \bar{x}_{11}) + (\Delta x_{24} \bar{x}_{23} - \Delta x_{22} \bar{x}_{21}) + (d_5(l_{1i}) \bar{x}_{33} - d_5(0) \bar{x}_{31}) \} \\ &\quad + \sum_{k=1}^3 \{ \Delta F_k(l_{1i}) \bar{x}_{k3} - \Delta F_k(0) \bar{x}_{k1} \} \end{aligned} \quad (3.41)$$

$$\begin{aligned} EI \Delta \mathbf{r}''_h \cdot \bar{\mathbf{r}}'_h \Big|_0^{l_{1i}} &= EI (\Delta x''_{1h} \bar{x}'_{1h} + \Delta x''_{2h} \bar{x}'_{2h} + d_7 \bar{x}'_{3h}) \Big|_0^{l_{1i}} \\ &= [EI \{ \sum_{k=1}^2 \left(\sum_{i=1}^4 \sum_{j=1}^4 N''_i N'_j \Delta x_{ki} \bar{x}_{kj} \right) + d_7 \left(\sum_{i=1}^4 N'_i \bar{x}_{3i} \right) \}] \Big|_0^{l_{1i}} \\ &= (\Delta M_1(l_{1i}) \bar{x}_{14} - \Delta M_1(0) \bar{x}_{12}) + (\Delta M_2(l_{1i}) \bar{x}_{24} - \Delta M_2(0) \bar{x}_{22}) \\ &\quad + (\Delta M_3(l_{1i}) \bar{x}_{34} - \Delta M_3(0) \bar{x}_{32}) \end{aligned} \quad (3.42)$$

3.4 Matrix Dynamic Equilibrium Equation

In previous section, The finite element representations for each individual component of the incremental weak form have been given in the series form of Equations (3.30) - (3.42) through the incorporation of Hermite polynomials selected as the shape or basis functions. A total of 12 degrees of freedom per element (six for each node) have been used for the representation of the deformation, namely the deflections and their derivatives (rotations).

To determine the deflections and the rotations at node $(x_{j1}, x_{j3}$ and x_{j2}, x_{j4} , $j = 1, 2, 3$) of coefficients that will characterize the approximate solution in (3.27), we substitute (3.30) through (3.42) into (3.21) to obtain the condition

$$\begin{aligned}
 & \sum_{k=1}^3 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{l_1} m_i N_i N_j ds_1 \right) \Delta \ddot{x}_{kj} \right\} \bar{x}_{ki} \right] \\
 & + \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{l_1} 2m_f V_1 N_i N_j' ds_1 \right) \Delta \dot{x}_{kj} \right\} \bar{x}_{ki} \right] + \sum_{i=1}^4 \left(\int_0^{l_1} 2m_f V_1 d_6 N_i ds_1 \right) \bar{x}_{3i} \\
 & + \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{l_1} -m_f V_1^2 N_i' N_j' ds_1 \right) \Delta x_{kj} \right\} \bar{x}_{ki} \right] + \sum_{i=1}^4 \left(\int_0^{l_1} -m_f V_1^2 d_5 N_i' ds_1 \right) \bar{x}_{3i} \\
 & + \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{l_1} EI N_i'' N_j'' ds_1 \right) \Delta x_{kj} \right\} \bar{x}_{ki} \right] + \sum_{i=1}^4 \left(\int_0^{l_1} EI d_7 N_i'' ds_1 \right) \bar{x}_{3i} \\
 & + \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \sum_{j=1}^4 \left(\int_0^{l_1} (T_0 - EI \kappa^2) N_i' N_j' ds_1 \right) \Delta x_{kj} \right\} \bar{x}_{ki} \right] \\
 & + \sum_{i=1}^4 \left(\int_0^{l_1} (T_0 - EI \kappa^2) d_5 N_i' ds_1 \right) \bar{x}_{3i} \\
 & + \sum_{i=1}^4 \sum_{j=1}^4 \left\{ \left(\int_0^{l_1} F_1 x_{1h}' N_i' N_j' ds_1 \right) \Delta x_{1j} + \left(\int_0^{l_1} F_2 x_{1h}' N_i' N_j' ds_1 \right) \Delta x_{2j} \right\} \bar{x}_{1i} \\
 & + \sum_{i=1}^4 \sum_{j=1}^4 \left\{ \left(\int_0^{l_1} F_1 x_{2h}' N_i' N_j' ds_1 \right) \Delta x_{1j} + \left(\int_0^{l_1} F_2 x_{2h}' N_i' N_j' ds_1 \right) \Delta x_{2j} \right\} \bar{x}_{2i}
 \end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^4 \sum_{j=1}^4 \{ (\int_0^{l_1} F_1 d_1 N'_i N'_j ds_1) \Delta x_{1j} + (\int_0^{l_1} F_2 d_1 N'_i N'_j ds_1) \Delta x_{2j} \} \bar{x}_{3i} \\
& + \sum_{i=1}^4 (\int_0^{l_1} F_3 d_5 x'_{1h} N'_i ds_1) \bar{x}_{1i} + \sum_{i=1}^4 (\int_0^{l_1} F_3 d_5 x'_{2h} N'_i ds_1) \bar{x}_{2i} \\
& + \sum_{i=1}^4 (\int_0^{l_1} F_3 d_5 d_1 N'_i ds_1) \bar{x}_{3i} \\
& + \sum_{i=1}^4 \sum_{j=1}^4 \{ (\int_0^{l_1} -2EI x'_{1h} x''_{1h} N'_i N'_j ds_1) \Delta x_{1j} + (\int_0^{l_1} -2EI x'_{1h} x''_{2h} N'_i N'_j ds_1) \Delta x_{2j} \} \bar{x}_{1i} \\
& + \sum_{i=1}^4 \sum_{j=1}^4 \{ (\int_0^{l_1} -2EI x'_{2h} x''_{1h} N'_i N'_j ds_1) \Delta x_{1j} + (\int_0^{l_1} -2EI x'_{2h} x''_{2h} N'_i N'_j ds_1) \Delta x_{2j} \} \bar{x}_{2i} \\
& + \sum_{i=1}^4 \sum_{j=1}^4 \{ (\int_0^{l_1} -2EI x''_{1h} d_1 N'_i N'_j ds_1) \Delta x_{1j} + (\int_0^{l_1} -2EI x''_{2h} d_1 N'_i N'_j ds_1) \Delta x_{2j} \} \bar{x}_{3i} \\
& + \sum_{i=1}^4 (\int_0^{l_1} -2EI d_3 d_7 x'_{1h} N'_i ds_1) \bar{x}_{1i} + \sum_{i=1}^4 (\int_0^{l_1} -2EI d_3 d_7 x'_{2h} N'_i ds_1) \bar{x}_{2i} \\
& + \sum_{i=1}^4 (\int_0^{l_1} -2EI d_1 d_3 d_7 N'_i ds_1) \bar{x}_{3i} \\
& + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} H d_3 N'_i N'_j ds_1) \Delta x_{2j} \bar{x}_{1i} + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} -H d_3 N'_i N'_j ds_1) \Delta x_{1j} \bar{x}_{2i} \\
& + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} H x''_{2h} N'_i N'_j ds_1) \Delta x_{1j} \bar{x}_{3i} + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} -H x'_{1h} N'_i N'_j ds_1) \Delta x_{2j} \bar{x}_{3i} \\
& + \sum_{i=1}^4 (\int_0^{l_1} -H d_5 x''_{2h} N'_i ds_1) \bar{x}_{1i} + \sum_{i=1}^4 (\int_0^{l_1} H d_5 x'_{1h} N'_i ds_1) \bar{x}_{2i} \\
& + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} -H d_1 N'_i N'_j ds_1) \Delta x_{2j} \bar{x}_{1i} + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} H d_1 N'_i N'_j ds_1) \Delta x_{1j} \bar{x}_{2i} \\
& + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} H x'_{1h} N'_i N'_j ds_1) \Delta x_{2j} \bar{x}_{3i} + \sum_{i=1}^4 \sum_{j=1}^4 (\int_0^{l_1} -H x'_{2h} N'_i N'_j ds_1) \Delta x_{1j} \bar{x}_{3i} \\
& + \sum_{i=1}^4 (\int_0^{l_1} H d_7 x'_{2h} N'_i ds_1) \bar{x}_{1i} + \sum_{i=1}^4 (\int_0^{l_1} -H d_7 x'_{1h} N'_i ds_1) \bar{x}_{2i}
\end{aligned}$$

$$\begin{aligned}
&= \sum_{k=1}^3 \sum_{i=1}^4 \left(\int_0^{l_1} \Delta q_k N_i ds_1 \right) x_{ki} \\
&+ \sum_{k=1}^2 \left[\sum_{i=1}^4 \left\{ \int_0^{l_1} -(\Delta F_1 x'_{1h} + \Delta F_2 x'_{2h} + \Delta F_3 d_1) x'_{kh} N'_i ds_1 \right\} \bar{x}_{ki} \right] \\
&+ \sum_{i=1}^4 \left\{ \int_0^{l_1} -(\Delta F_1 x'_{1h} + \Delta F_2 x'_{2h} + \Delta F_3 d_1) d_1 N'_i ds_1 \right\} \bar{x}'_{3i} \\
&- m_f V_1^2 \{ (\Delta x_{14} \bar{x}_{13} - \Delta x_{12} \bar{x}_{11}) + (\Delta x_{24} \bar{x}_{23} - \Delta x_{22} \bar{x}_{21}) + (d_5(l_{1i}) \bar{x}_{33} - d_5(0) \bar{x}_{31}) \} \\
&+ \sum_{k=1}^3 \{ \Delta F_k(l_{1i}) \bar{x}_{k3} - \Delta F_k(0) \bar{x}_{k1} \} \\
&+ (\Delta M_1(l_{1i}) \bar{x}_{14} - \Delta M_1(0) \bar{x}_{12}) + (\Delta M_2(l_{1i}) \bar{x}_{24} - \Delta M_2(0) \bar{x}_{22}) \\
&+ (\Delta M_3(l_{1i}) \bar{x}_{34} - \Delta M_3(0) \bar{x}_{32}) \quad (3.43)
\end{aligned}$$

Factorizing the coefficients of test function ($\bar{x}_{k1}$, $\bar{x}_{k2}$, and $\bar{x}_{k3}$, $k = 1, 2, 3$ from Eq. (3.43), we may have the following compact form

$$\sum_{k=1}^3 \left[\sum_{i=1}^4 \bar{x}_{ki} \left\{ \sum_{j=1}^4 (M_{ij} \Delta \ddot{x}_{kj} + C_{ij} \Delta \dot{x}_{kj} + K_{ij} \Delta x_{kj}) \right\} \right] = \sum_{k=1}^3 \sum_{i=1}^4 (\Delta f_{ki} + \Delta F_{ki}) \bar{x}_k \quad (3.44)$$

Since the $\bar{x}_{ki}$ are arbitrary, (3.44) represents 12 independent equations to be satisfied by the Δx_{kj} rather than the single equation it may appear to be. In other words, the coefficients of test function may be taken out by the natural choices for the set of $\bar{x}_{ki}$. For an instance, for the choice of $\bar{x}_{11} = 1$, $\bar{x}_{12} = \bar{x}_{13} = \bar{x}_{14} = \dots = \bar{x}_{33} = \bar{x}_{34} = 0$ we have

$$\sum_{j=1}^4 (M_{1j} \Delta \ddot{x}_{1j} + C_{1j} \Delta \dot{x}_{1j} + K_{1j} \Delta x_{1j}) \} = \sum_{j=1}^4 (\Delta f_{1j} + \Delta F_{1j}) \bar{x}_{1j} \quad (3.45)$$

Continuing this way, we arrive at the system of 12 linear independent equations in the 12 unknown coefficients of approximate solution, namely Δx_{kj} , $k = 1, 2, 3$, and $j = 1, 2, 3, 4$.

Writing it in matrix form,

$$\begin{aligned}
 & \begin{bmatrix} [M_{ij}] & [0] & [0] \\ [0] & [M_{ij}] & [0] \\ [0] & [0] & [M_{ij}] \end{bmatrix} \begin{Bmatrix} \{\Delta \dot{x}_{1j}\} \\ \{\Delta \dot{x}_{2j}\} \\ \{\Delta \dot{x}_{3j}\} \end{Bmatrix} + \begin{bmatrix} [C_{ij}] & [0] & [0] \\ [0] & [C_{ij}] & [0] \\ [0] & [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\Delta \dot{x}_{1j}\} \\ \{\Delta \dot{x}_{2j}\} \\ \{\Delta \dot{x}_{3j}\} \end{Bmatrix} + \begin{Bmatrix} \{0\} \\ \{0\} \\ \{f_{ej}\} \end{Bmatrix} \\
 & + \begin{bmatrix} [K_{1ij} + K_{2ij} + K_{3ij} + K_{4ij}^{11} + K_{5ij}^{11}] & [K_{4ij}^{12} + K_{5ij}^{12} + K_{6ij}^{12} + K_{7ij}^{12}] & [0] \\ [K_{4ij}^{21} + K_{5ij}^{21} + K_{6ij}^{21} + K_{7ij}^{21}] & [K_{1ij} + K_{2ij} + K_{3ij} + K_{4ij}^{22} + K_{5ij}^{22}] & [0] \\ [K_{4ij}^{31} + K_{5ij}^{31} + K_{6ij}^{31} + K_{7ij}^{31}] & [K_{4ij}^{32} + K_{5ij}^{32} + K_{6ij}^{32} + K_{7ij}^{32}] & [0] \end{bmatrix} \begin{Bmatrix} \{\Delta x_{1j}\} \\ \{\Delta x_{2j}\} \\ \{\Delta x_{3j}\} \end{Bmatrix} \\
 & + \begin{Bmatrix} \{f_{k4j}^1 + f_{k5j}^1 + f_{k6j}^1 + f_{k7j}^1\} \\ \{f_{k4j}^2 + f_{k5j}^2 + f_{k6j}^2 + f_{k7j}^2\} \\ \{f_{k1j}^3 + f_{k2j}^3 + f_{k3j}^3 + f_{k4j}^3 + f_{k5j}^3\} \end{Bmatrix} = \begin{Bmatrix} \{q_j^1 + f_{fj}^1\} \\ \{q_j^2 + f_{fj}^2\} \\ \{q_j^3 + f_{fj}^3\} \end{Bmatrix} + \begin{bmatrix} [C_{ej}] & [0] & [0] \\ [0] & [C_{ej}] & [0] \\ [0] & [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\Delta x_{1j}\} \\ \{\Delta x_{2j}\} \\ \{\Delta x_{3j}\} \end{Bmatrix} \\
 & + \begin{Bmatrix} \{0\} \\ \{0\} \\ \{f_{cej}\} \end{Bmatrix} + \begin{Bmatrix} \{\Delta F_{1j}\} \\ \{\Delta F_{2j}\} \\ \{\Delta F_{3j}\} \end{Bmatrix} \tag{3.46}
 \end{aligned}$$

where,

$$\begin{aligned}
 M_{ij} &= \int_0^{l_1} m_i N_i N_j ds_1, & C_{ij} &= \int_0^{l_1} 2m_f V_1 N_i N_j' ds_1 \\
 f_{ej} &= \int_0^{l_1} 2m_f V_1 d_6 N_j ds_1, & K_{1ij} &= \int_0^{l_1} -m_f V_1^2 N_i' N_j' ds_1 \\
 f_{k1j}^3 &= \int_0^{l_1} -m_f V_1^2 ds_1 N_j' ds_1, & K_{2ij} &= \int_0^{l_1} EI N_i'' N_j'' ds_1 \\
 f_{k2j}^3 &= \int_0^{l_1} EI d_7 N_j'' ds_1, & K_{3ij} &= \int_0^{l_1} (T_e - EI \kappa^2) N_i' N_j' ds_1 \\
 f_{k3j}^3 &= \int_0^{l_1} (T_e - EI \kappa^2) ds_1 N_j' ds_1, & K_{4ij}^{11} &= \int_0^{l_1} F_1 x_{1h}' N_i' N_j' ds_1 \\
 K_{4ij}^{12} &= \int_0^{l_1} F_2 x_{1h}' N_i' N_j' ds_1, & K_{4ij}^{21} &= \int_0^{l_1} F_1 x_{2h}' N_i' N_j' ds_1
 \end{aligned}$$

$$K_{4ij}^{22} = \int_0^{l_1} F_1 x'_{2h} N'_i N'_j ds_1,$$

$$K_{4ij}^{32} = \int_0^{l_1} F_2 d_1 N'_i N'_j ds_1,$$

$$f_{k4j}^2 = \int_0^{l_1} F_3 d_5 x'_{2h} N'_j ds_1,$$

$$K_{5ij}^{11} = \int_0^{l_1} -2EI x'_{1h} x''_{1h} N'_i N''_j ds_1,$$

$$K_{5ij}^{21} = \int_0^{l_1} -2EI x'_{2h} x''_{1h} N'_i N''_j ds_1,$$

$$K_{5ij}^{31} = \int_0^{l_1} -2EI x''_{1h} d_1 N'_i N''_j ds_1,$$

$$f_{k5j}^1 = \int_0^{l_1} -2EI d_3 d_7 x'_{1h} N'_j ds_1,$$

$$f_{k5j}^3 = \int_0^{l_1} -2EI d_1 d_3 d_7 N'_j ds_1,$$

$$K_{6ij}^{21} = \int_0^{l_1} -H d_3 N'_i N'_j ds_1,$$

$$K_{6ij}^{32} = \int_0^{l_1} -H x''_{1h} N'_i N'_j ds_1,$$

$$f_{k6j}^2 = \int_0^{l_1} H d_5 x''_{1h} N'_j ds_1,$$

$$K_{7ij}^{21} = \int_0^{l_1} H d_1 N'_i N''_j ds_1,$$

$$K_{7ij}^{32} = \int_0^{l_1} -H x'_{2h} N'_i N''_j ds_1,$$

$$K_{4ij}^{31} = \int_0^{l_1} F_1 d_1 N'_i N'_j ds_1$$

$$f_{k4j}^1 = \int_0^{l_1} F_3 d_5 x'_{1h} N'_j ds_1$$

$$f_{k4j}^3 = \int_0^{l_1} F_3 d_5 d_1 N'_j ds_1$$

$$K_{5ij}^{12} = \int_0^{l_1} -2EI x'_{1h} x''_{2h} N'_i N''_j ds_1$$

$$K_{5ij}^{22} = \int_0^{l_1} -2EI x'_{2h} x''_{2h} N'_i N''_j ds_1$$

$$K_{5ij}^{32} = \int_0^{l_1} -2EI x''_{2h} d_1 N'_i N''_j ds_1$$

$$f_{k5j}^2 = \int_0^{l_1} -2EI d_3 d_7 x'_{2h} N'_j ds_1$$

$$K_{6ij}^{12} = \int_0^{l_1} H d_3 N'_i N'_j ds_1$$

$$K_{6ij}^{31} = \int_0^{l_1} H x''_{2h} N'_i N'_j ds_1$$

$$f_{k6j}^1 = \int_0^{l_1} -H d_5 x''_{2h} N'_j ds_1$$

$$K_{7ij}^{12} = \int_0^{l_1} -H d_1 N'_i N''_j ds_1$$

$$K_{7ij}^{31} = \int_0^{l_1} H x'_{1h} N'_i N''_j ds_1$$

$$f_{k7j}^1 = \int_0^{l_1} H d_7 x'_{2h} N'_j ds_1$$

$$\begin{aligned}
f_{kj}^2 &= \int_0^{l_1} -H d_7 x'_{1h} N'_j ds_1, & q_j^1 &= \int_0^{l_1} \Delta q_1 N_j ds_1 \\
q_j^2 &= \int_0^{l_1} \Delta q_2 N_j ds_1, & q_j^3 &= \int_0^{l_1} \Delta q_1 N_j ds_1 \\
f_{jj}^1 &= \int_0^{l_1} -(\Delta F_1 x'_{1h} + \Delta F_2 x'_{2h} + \Delta F_3 d_1) x'_{1h} N'_j ds_1 \\
f_{jj}^2 &= \int_0^{l_1} -(\Delta F_1 x'_{1h} + \Delta F_2 x'_{2h} + \Delta F_3 d_1) x'_{2h} N'_j ds_1 \\
f_{jj}^3 &= \int_0^{l_1} -(\Delta F_1 x'_{1h} + \Delta F_2 x'_{2h} + \Delta F_3 d_1) d_1 N'_j ds_1 \quad (3.47)
\end{aligned}$$

$$\begin{aligned}
\{\Delta F_{1j}\} &= \begin{Bmatrix} -\Delta F_1(0) \\ -\Delta M_1(0) \\ \Delta F_1(l_{1i}) \\ \Delta M_1(l_{1i}) \end{Bmatrix} & \{\Delta F_{2j}\} &= \begin{Bmatrix} -\Delta F_2(0) \\ -\Delta M_2(0) \\ \Delta F_2(l_{2i}) \\ \Delta M_2(l_{2i}) \end{Bmatrix} & \{\Delta F_{3j}\} &= \begin{Bmatrix} -\Delta F_3(0) \\ -\Delta M_3(0) \\ \Delta F_3(l_{3i}) \\ \Delta M_3(l_{3i}) \end{Bmatrix} \\
[C_{elj}] &= -m_f V_I^2 \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \{f_{cej}\} &= -m_f V_I^2 \begin{Bmatrix} -d_5(0) \\ 0 \\ -d_5(l_{1i}) \\ 0 \end{Bmatrix} \quad (3.48)
\end{aligned}$$

In equation (3.46), the subscript i and j run from 1 to 4 and a total 12 degrees of freedom per element are used as shown in Figure 3.2.

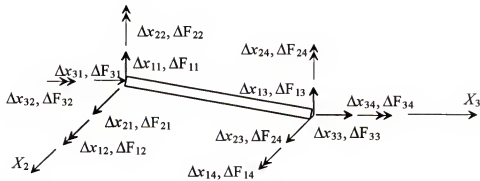


Figure 3.2: An Element of Riser Showing Forces and Displacements at the Node

Also we may represent equation (3.46) in the most compact form:

$$[M]\{\Delta \ddot{x}\} + [C]\{\Delta \dot{x}\} + [K]\{\Delta x\} = \{\Delta f\} + \{\Delta F\} \quad (3.49)$$

where,

$$[M] = \begin{bmatrix} [M_{ij}] & [0] & [0] \\ [0] & [M_{ij}] & [0] \\ [0] & [0] & [M_{ij}] \end{bmatrix} \quad [C] = \begin{bmatrix} [C_{ij}] & [0] & [0] \\ [0] & [C_{ij}] & [0] \\ [0] & [0] & [0] \end{bmatrix}$$

$$[K] = \begin{bmatrix} [K_{1ij} + K_{2ij} + K_{3ij} + K_{4ij}^{11} + K_{5ij}^{11}] & [K_{4ij}^{12} + K_{5ij}^{12} + K_{6ij}^{12} + K_{7ij}^{12}] & [0] \\ [K_{4ij}^{21} + K_{5ij}^{21} + K_{6ij}^{21} + K_{7ij}^{21}] & [K_{1ij} + K_{2ij} + K_{3ij} + K_{4ij}^{22} + K_{5ij}^{22}] & [0] \\ [K_{4ij}^{31} + K_{5ij}^{31} + K_{6ij}^{31} + K_{7ij}^{31}] & [K_{4ij}^{32} + K_{5ij}^{32} + K_{6ij}^{32} + K_{7ij}^{32}] & [0] \end{bmatrix}$$

$$- \begin{bmatrix} [C_{eij}] & [0] & [0] \\ [0] & [C_{eij}] & [0] \\ [0] & [0] & [0] \end{bmatrix}$$

$$\{\Delta f\} = \left\{ \begin{bmatrix} q_j^1 + f_{jj}^1 \\ q_j^2 + f_{jj}^2 \\ q_j^3 + f_{jj}^3 \end{bmatrix} \right\} - \left\{ \begin{bmatrix} f_{k4j}^1 + f_{k5j}^1 + f_{k6j}^1 + f_{k7j}^1 \\ f_{k4j}^2 + f_{k5j}^2 + f_{k6j}^2 + f_{k7j}^2 \\ f_{k1j}^3 + f_{k2j}^3 + f_{k3j}^3 + f_{k4j}^3 + f_{k5j}^3 \end{bmatrix} \right\} - \left\{ \begin{bmatrix} \{0\} \\ \{0\} \\ \{f_{ej}\} \end{bmatrix} \right\} + \left\{ \begin{bmatrix} \{0\} \\ \{0\} \\ \{f_{cej}\} \end{bmatrix} \right\}$$

$$\{\Delta F\} = \left\{ \begin{bmatrix} \{\Delta F_{1j}\} \\ \{\Delta F_{2j}\} \\ \{\Delta F_{3j}\} \end{bmatrix} \right\} \quad (3.50)$$

In (3.49), the matrix $[M]$ is a consistent mass matrix which is identical to the one derived for small deformation models and includes the added mass of the riser and the mass of the drilling fluid. $[C]$ is an anti-symmetric matrix due to the coriolis force resulting from the internal flow. Furthermore $[K]$ is the time and deformation dependent stiffness matrix and consists of 8 components: $[K_1]$ is due to the centrifugal force from the internal flowing fluid, $[K_2]$ is the bending stiffness matrix, and $[K_3]$ is the geometric stiffness matrix which implicates the nonlinear coupling between axial tension and

bending. The previous three matrixes are symmetric, while $[K_4]$, $[K_5]$, $[K_6]$ and $[K_7]$ are non-symmetric, full matrixes. $[K_4]$, $[K_5]$ are due to the variable longitudinal tension and $[K_6]$, $[K_7]$ are due to the torque H. Finally, $[C_e]$ is the centrifugal force component at the both end of element. As forcing components, $\{f\}$ is the deformation dependent equivalent nodal force vector and $\{F\}$ is the internal force and moment vector.

As stated in section 2.5.1, utilization of the kinematic constraint on unit tangent vector leads to the reduction of the degrees of freedom per element from 12 to 8, that is, the vertical degrees of freedom can be computed in terms of the lateral ones. Thus, we may need to evaluate the lateral degrees from Eq. (3.46). Manipulating (3.46) through the partition of matrix and rewriting only in terms of lateral degrees,

$$\begin{aligned}
 & \begin{bmatrix} [M_{ij}] & [0] \\ [0] & [M_{ij}] \end{bmatrix} \begin{Bmatrix} \{\Delta \ddot{x}_{1j}\} \\ \{\Delta \ddot{x}_{2j}\} \end{Bmatrix} + \begin{bmatrix} [C_{ij}] & [0] \\ [0] & [C_{ij}] \end{bmatrix} \begin{Bmatrix} \{\Delta \dot{x}_{1j}\} \\ \{\Delta \dot{x}_{2j}\} \end{Bmatrix} \\
 & + \begin{bmatrix} [K_{1ij} + K_{2ij} + K_{3ij} + K_{4ij}^{11} + K_{5ij}^{11} - C_{eij}] & [K_{4ij}^{12} + K_{5ij}^{12} + K_{6ij}^{12} + K_{7ij}^{12}] \\ [K_{4ij}^{21} + K_{5ij}^{21} + K_{6ij}^{21} + K_{7ij}^{21}] & [K_{1ij} + K_{2ij} + K_{3ij} + K_{4ij}^{22} + K_{5ij}^{22} - C_{eij}] \end{bmatrix} \begin{Bmatrix} \{\Delta x_{1j}\} \\ \{\Delta x_{2j}\} \end{Bmatrix} \\
 & = \begin{Bmatrix} \{q_j^1 + f_{jj}^1\} \\ \{q_j^2 + f_{jj}^2\} \end{Bmatrix} - \begin{Bmatrix} \{f_{k4j}^1 + f_{k5j}^1 + f_{k6j}^1 + f_{k7j}^1\} \\ \{f_{k4j}^2 + f_{k5j}^2 + f_{k6j}^2 + f_{k7j}^2\} \end{Bmatrix} + \begin{Bmatrix} \{\Delta F_{1j}\} \\ \{\Delta F_{2j}\} \end{Bmatrix} \quad (3.51)
 \end{aligned}$$

For the vertical degrees of freedom, the matrix equilibrium can be written as:

$$\begin{aligned}
 & [M_{ij}] \{\Delta \ddot{x}_{3j}\} + \{f_{ej}\} + \begin{bmatrix} [K_{4ij}^{31} + K_{5ij}^{31} + K_{6ij}^{31} + K_{7ij}^{31}] & [K_{4ij}^{32} + K_{5ij}^{32} + K_{6ij}^{32} + K_{7ij}^{32}] \end{bmatrix} \begin{Bmatrix} \{\Delta x_{1j}\} \\ \{\Delta x_{2j}\} \end{Bmatrix} \\
 & + \{f_{k1j}^3 + f_{k2j}^3 + f_{k3j}^3 + f_{k4j}^3 + f_{k5j}^3\} = \{q_j^3 + f_{jj}^3\} + \{f_{ej}\} + \{\Delta F_{3j}\} \quad (3.52)
 \end{aligned}$$

Since the vibrations in vertical direction are not important, we can ignore the inertia term in (3.52) and obtain the internal forces and moment from (3.52) as following:

$$\begin{aligned} \{\Delta F_{3j}\} = & -\{f_{ej}\} + \{f_{k1j}^3 + f_{k2j}^3 + f_{k3j}^3 + f_{k4j}^3 + f_{k5j}^3\} - \{q_j^3 + f_{fj}^3\} + \{f_{ej}\} \\ & + \left[\left[K_{4ij}^{31} + K_{5ij}^{31} + K_{6ij}^{31} + K_{7ij}^{31} \right] \left[K_{4ij}^{32} + K_{5ij}^{32} + K_{6ij}^{32} + K_{7ij}^{32} \right] \right] \begin{Bmatrix} \{\Delta x_{1j}\} \\ \{\Delta x_{2j}\} \end{Bmatrix} \end{aligned} \quad (3.53)$$

Having calculated the matrixes and equations describing our approximation over each finite element, the next step is to assemble the equations describing the approximation on the entire mesh by adding up the contributions to these equations furnished by each element. This assembling procedure is omitted because it is well known and referenced in several books and papers.

3.5 Finite Element Model of Linear System

This section formulates the numerical model of linear governing equations derived under the assumption of small deformation, that is, transverse vibration and in-line dynamic response of current-vortex model are modeled using finite element method. The same procedure as discussed in previous sections (section 3.1 through 3.4) is applied except the utilization of the incremental operator. The application of the incremental operator is not necessary in this problem because the inaccuracy of solution or the inefficiency in convergence is not expected.

The mathematical model of current induced vibration can be obtained simply by combining Eq. (2.69), (2.72) and (2.75):

$$\begin{aligned} m_i \ddot{x}_1 + 2m_f N_1 \dot{x}_1' + m_f N_1^2 x_1'' + (Elx_1'')'' - wx_1' - T_e x_1'' \\ = a_4 \rho_w D_o u_e (\dot{w} - \dot{x}_1) p(x_3) - \frac{1}{2} \rho_w u_e D_o C_D \dot{x}_1 (1 - p(x_3)) \end{aligned} \quad (3.54)$$

$$\ddot{w} + \omega_s^2 w = (a_1' - a_4') u_e \dot{w} / D_o - a_2' \dot{w}^3 / (u_e D_o) + a_4' u_e \dot{x}_1 / D_o \quad (3.55)$$

$$m_i \ddot{x}_2 + 2m_f N_1 \dot{x}_2' + m_f N_1^2 x_2'' + (Elx_2'')'' - wx_2' - T_e x_2'' = q_2 \quad (3.56)$$

As we discussed in chapter 2, those three equations represent the transverse vortex induced vibration, the fluid oscillator equation about the hidden flow variable w , and the riser vibration in the current direction. The forcing term q_2 in the right side of Eq. (3.56) can be derived by combining Eq. (2.38), (2.39), and (2.48), and only taking the drag term due to current and given as,

$$q_2 = -\frac{1}{2} \rho_w C_D D_o |\dot{x}_2 - u_c| (\dot{x}_2 - u_c) \quad (3.57)$$

The weak form of the equations are produced in multiplying the residual by a sufficiently smooth test function, integrating the product by parts, and equating the result to zero. The details in derivation are given in Appendix B and only the results are written here as followings:

$$\begin{aligned} \int_0^l [m_i \ddot{x}_1 \bar{x}_1 + 2m_f V_1 \dot{x}_1' \bar{x}_1 - m_f V_1^2 x_1' \bar{x}_1 + (EI x_1'') \bar{x}_1'' + (T_e x_1') \bar{x}_1' - a_4 \rho_w D_o u_c p(x_3) \dot{w} \bar{x}_1 \\ + \{a_4 \rho_w D_o u_c p(x_3) + \frac{1}{2} \rho_w u_c D_o C_D (1 - p(x_3))\} \dot{x}_1 \bar{x}_1] dx_3 + m_f V_1^2 x_1' \bar{x}_1 \Big|_0^l \\ + \{-(T_e x_1') \bar{x}_1 + (EI x_1'')' \bar{x}_1 - (EI x_1'') \bar{x}_1'\} \Big|_0^l = 0 \end{aligned} \quad (3.58)$$

$$\begin{aligned} \int_0^l \{\dot{w} \bar{w} + (a_4' - a_1') (u_c / D_o) \dot{w} \bar{w} + \omega_s^2 w \bar{w} + (a_2' / (u_c D_o)) \dot{w}^3 \bar{w} \\ - a_4' u_c \dot{x}_1 / D_o \bar{w}\} dx_3 = 0 \end{aligned} \quad (3.59)$$

$$\begin{aligned} \int_0^l [m_i \ddot{x}_2 \bar{x}_2 + 2m_f V_1 \dot{x}_2' \bar{x}_2 - m_f V_1^2 x_2' \bar{x}_2 + (EI x_2'') \bar{x}_2'' + (T_e x_2') \bar{x}_2' \\ - \frac{1}{2} \rho_w C_D D_o |u_c - \dot{x}_2| (u_c - \dot{x}_2) \bar{x}_2] dx_3 \\ + \{m_f V_1^2 x_2' \bar{x}_2 - (T_e x_2') \bar{x}_1 + (EI x_1'')' \bar{x}_1 - (EI x_1'') \bar{x}_1'\} \Big|_0^l = 0 \end{aligned} \quad (3.60)$$

Implementation of Hermite polynomials as basis functions yields the matrix dynamic equilibrium equations constructed in terms of the unknown deformations at node. For the construction of the matrix equation, the same processes applied in section 3.3 and

3.4 are used. The details in derivation are given in Appendix B and the resulting matrix equations are:

$$[\hat{M}_{ij}]\{\ddot{x}_{1j}(t)\} + [\hat{C}_{ij} + \hat{C}_{fij}]\{\dot{x}_{1j}(t)\} + [\hat{T}_{ij} - \hat{C}_{eij} + \hat{K}_{ij}]\{x_{1j}(t)\} = [\hat{F}_{ij}]\{\dot{w}_j(t)\} \quad (3.61)$$

$$[\bar{M}_{ij}]\{\ddot{w}_j(t)\} + [\bar{C}_{ij}]\{\dot{w}_j(t)\} + [\bar{K}_{ij}]\{w_j(t)\} = [\bar{F}_{ij}]\{\dot{x}_{ij}\} - \bar{f}_j \quad (3.62)$$

$$[\tilde{M}_{ij}]\{\ddot{x}_{2j}(t)\} + [\tilde{C}_{ij}]\{\dot{x}_{2j}(t)\} + [\tilde{T}_{ij} - \tilde{C}_{eij} + \tilde{K}_{ij}]\{x_{2j}(t)\} = \tilde{f}_j \quad (3.63)$$

where,

$$\begin{aligned} \hat{M}_{ij} &= \tilde{M}_{ij} = \int_{\Omega_e} m_i N_i N_j dx_3, & \hat{C}_{ij} &= \tilde{C}_{ij} = \int_{\Omega_e} 2m_f V_1 N_i N_j' dx_3 \\ \hat{C}_{fij} &= \int_{\Omega_e} \{a_4' \rho_w D_o u_c p(x_3) + \frac{1}{2} \rho_w u_c D_o C_D (1-p(x_3))\} N_i N_j dx_3 \\ \hat{T}_{ij} &= \tilde{T}_{ij} = \int_{\Omega_e} T_c N_i' N_j' dx_3, & \hat{C}_{eij} &= \tilde{C}_{eij} = \int_{\Omega_e} m_f V_1^2 N_i' N_j' dx_3 \\ \hat{K}_{ij} &= \tilde{K}_{ij} = \int_{\Omega_e} EI N_i'' N_j'' dx_3, & \hat{F}_{ij} &= \int_{\Omega_e} a_4' \rho_w D_o u_c p(x_3) N_i N_j dx_3 \\ f_j &= \int_{\Omega_e} \frac{1}{2} \rho_w D_o C_D |u_c - \dot{x}_{2h}| (u_c - \dot{x}_{2h}) N_j dx_3 \\ \bar{M}_{ij} &= \int_{\Omega_e} N_i N_j dx_3, & \bar{C}_{ij} &= \int_{\Omega_e} (a_4' - a_1') (u_c / D_o) N_i N_j dx_3 \\ \bar{K}_{ij} &= \int_{\Omega_e} \omega_s^2 N_i N_j dx_3, & \bar{F}_{ij} &= \int_{\Omega_e} a_4' (u_c / D_o) N_i N_j dx_3 \\ \bar{f}_j &= \int_{\Omega_e} \{a_2' / (u_c D_o)\} \dot{w}_h^3 N_j dx_3 \end{aligned} \quad (3.64)$$

Eqs. (3.61) and (3.62) represent the vortex induced vibration which characterizes self-excited oscillator. Once the forcing due to well-formed vorticities acts on riser, the velocity of riser itself act again to fluid particle as a forcing. Eq. (3.63) presents in-line vibration due to current. The characteristics of the coefficient matrix in (3.64) will be discussed in section 6.3.

CHAPTER 4

DEVELOPMENT OF A FINITE ELEMENT PROGRAM

In this chapter, we discuss the development of a finite element code for the solution of our two-point boundary value problem. Some considerations of code design are presented and overall structure of the code is outlined. However, it should be noted that the time step integration and the predictor-corrector scheme for the convergence in solution due to the nonlinear terms (subroutine NEWMRK) is not described in this chapter but treated in chapter 6.

4.1 Description of CODE

We now describe the features of our finite element code, called CODE which is designed to perform the finite element analysis of our two-point boundary value problem. Details on the construction of CODE and the various subroutines it contains are discussed in the remainder of this chapter. As viewed in Figure 4.1, CODE consists of the two functional units. Since the finite element modeling is defined by the input data, the reading and generation of the arrays of data are performed by the preprocessing unit of the program which consists of several subroutines. The actual finite element calculations

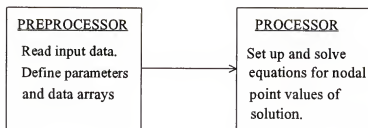


Figure 4.1: Functional Units of the Finite Element Code

are carried out in the processor. In this part of program, element matrices and vectors are calculated and assembled; boundary conditions are imposed, and a set of global equations is solved for the nodal-point values of solution.

The flowchart in Figure 4.2 shows each subroutine in CODE and its relation to other routines. Routine names are given in boxes and a brief description of their function is given in Figure 4.3. The data in CODE are organized into several labeled COMMON blocks, whose contents are summarized in Figure 4.4. As a general rule, the data are defined in routines called by the preprocessor PREP and used through CODE.

The main program serves only to call the routine SETINT that initializes numerical integration data (see Section 4.2) and the two main units of CODE as shown in Fig. 4.2.

4.2 Element Calculations

In the center of any finite element program, the calculation and assembly of the element matrixes and vectors are performed. These calculations consist of the integration indicated in the variational statement and are carried out in subroutine ELEM and ELEMV in conjunction with SETINT, SHAPE, GETCON, and GETMAT.

In order to compute the quantities of the integration in (3.47) or (3.64), we have to evaluate integrals of the form

$$\mathfrak{I} = \int_{x_1}^x f(x) dx \quad (4.1)$$

The numerical evaluation of the integral $\mathfrak{I}$ in (4.1) is accomplished by choosing a quadrature formula of the form

$$\mathfrak{I} \equiv I = \sum_{l=1}^{N_I} f(\bar{x}_l) w_l \quad (4.2)$$

where $\bar{x}_l$ are the integration points in the interval $x_1 \leq x \leq x_2$; the w_l are numbers, called

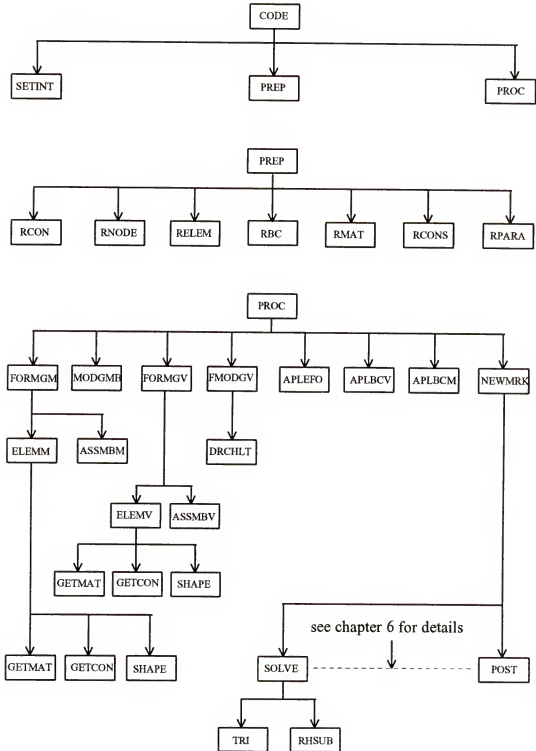


Figure 4.2: Flow Charts of Components of CODE

SETINT = Calculate quadrature rule data

PREP = Preprocessor: Calls routines to read and generate data

PROC = Processor: Forms and solves finite element equations

RCON = Reads control parameters

RNODE = Reads and generates nodal point coordinates

RELEM = Reads and generates element data

RBC = Reads boundary condition data

RMAT = Reads material property data

RCONS = Reads input constants and velocities

RPARA = Reads parameters for Newmark integration

FORMGM = Forms global coefficient matrices

MODGMB = Adds concentrated mass and damp at top

FORMGV = Forms global forcing vectors

FMODGV = Modifies global vectors for initial prescribed value at top and bottom
boundary

APLEFO = Adds point external forcing

APLBCV = Eliminates unnecessary forcing from global vector

APLBCM = Modifies global matrices for fixed boundary values

NEWMRK = Performs Newmark method for solving time series response.

See chapter 6 for details about algorithm, iterations, and predictor-corrector
scheme using extensibility condition for vertical degrees.

ELEMM = Calculates elemental coefficient matrices

ASSMBM = Adds element matrices to global matrices

ELEMV = Calculates elemental forcing vectors

ASSMBV = Adds element vectors to global vectors

GETMAT = Calculate material properties

GETCON = Gets input constants and parameters

SHAPE = Calculates shape function values

DRCHLT = Imposes boundary conditions

SOLVE = Solves the system of linear matrix equation for unknown displacement field

: } see chapter 6.

POST = Prints the required results

TRI = Performs Gaussian elimination

RHSUB = Performs elimination and back substitution on right hand side of equation

Figure 4.3: A Brief Description of Subroutines

CONTROL PARAMETERS

COMMON/CCON/NNODE,NELEM,NMAT,NCONS,NPOINT,NBW,NF

NNODE = number of node

NELEM = number of element

NMAT = number of materials

NCONS = number of characteristic constants

NPOINT = number of concentrated forces

NODAL POINT DATA

COMMON/CNODE/Z(I),X(I)

Z = coordinates of nodal points

X = displacements and rotations in horizontal direction.

ELEMENT DATA

COMMON/CELEM/KIND(I),MAT(I),NCON(I),NODES(J,I),NINT(I)

KIND = degree of polynomial in element shape function

MAT = number of material type of element

NCON = number of constant type of element

NODES = node numbers of nodes in element

NINT = number of integration points in element calculations

MATERIAL TYPE

COMMON/CMATL/PROP(I,J),CONS(I,J)

PROP = material properties

CONS = input constants and velocity

BOUNDARY CONDITION DATA

COMMON/KBC(J,I),VBC(J,I),KPOINT(I),POINT(J,I),NB(I,J)

KBC = boundary condition flag at each end

VBC = values of boundary condition data

KPOINT = node number of point loads

POINT = value of point loads,

NB = prescribed value flag; internally decided

Figure 4.4: Data Arrays in CODE

weights; and N_I is the order of formula. Again, we can replace these calculations as being performed on a master element $\tilde{\Omega}$ defined by the interval $-1 \leq \xi \leq 1$, so that limits of integration are -1 and 1. Specifically, if we choose as the transformation from ξ to x the relation

$$x = x_1 + \frac{1}{2}(x_2 - x_1)(1 + \xi) \quad (4.3)$$

then we have $x = x_1$ when $\xi = -1$, $x = x_2$ when $\xi = 1$, and $dx = (dx/d\xi) d\xi = \frac{1}{2}(x_2 - x_1) d\xi$. The form of the quadrature formula (4.2) now becomes

$$I = \frac{1}{2}(x_2 - x_1) \sum_{l=1}^{N_I} f(x(\xi_l)) w_l \quad (4.4)$$

The values of ξ_l , and w_l for $l = 1, \dots, N_I$ completely determine the numerical integration rule. In this problem, Gaussian quadrature rules are used. The Gauss rule of order N_I will integrate exactly polynomials of degree $2N_I - 1$. The integration such as (4.1) are performed in CODE using Gaussian quadrature of order 1,2,3, or 4. The data are stored in common block CINT by subroutine SETINT.

SHAPE which is called by ELEM and ELEMV calculates the values of the shape functions, their first and second derivatives with respect to the master element coordinate at a specified value of ξ . This routine is supplied with the degree of freedom in plane N and the value of ξ at which the values are to be calculated. The calculated values are returned in arrays PSI, DPSI, and DDPSI.

The evaluation of the integration of the form (4.4) is performed in ELEM and ELEMV. The values of the material properties and the input constants which contain in the integrands are supplied by GETMAT and GETCON. Since the integrand contains the first or second derivative with respect to the global coordinate x , we use the chain rule to calculate these quantities at the integration points. Thus,

$$\frac{dN_i}{dx} = \frac{d\tilde{N}_i}{d\xi} \frac{d\xi}{dx} = \frac{d\tilde{N}_i}{d\xi} \frac{2}{x_2 - x_1} \quad (4.5)$$

$$\frac{d^4 N_i}{dx^2} = \frac{d^2 \tilde{N}_i}{d\xi^2} \left(\frac{d\xi}{dx} \right)^2 = \frac{d^2 \tilde{N}}{d\xi^2} \left(\frac{2}{x_2 - x_1} \right)^2 \quad (4.6)$$

For instance, the explicit formulas for evaluating the integral multiplying the second derivative of shape function into its integrand can be evaluated as

$$\begin{aligned} \mathfrak{R} &= \int_{x_1}^x f(x) \frac{d^4 N_i}{dx^2} dx = \int_{-1}^1 f(x(\xi)) \frac{d^2 \tilde{N}}{d\xi^2} \left(\frac{2}{x_2 - x_1} \right)^2 \left(\frac{x_2 - x_1}{2} \right) d\xi \\ &= \sum_{i=1}^{N_k} f(x(\xi_i)) \frac{d^2 \tilde{N}}{d\xi^2} \left(\frac{2}{x_2 - x_1} \right)^2 \left(\frac{x_2 - x_1}{2} \right) w_i \end{aligned} \quad (4.7)$$

GETMAT and GETCON routine calculates the value of the material properties and input parameters at a given point respectively, and are called ELEM and ELEMV.

4.3 Preprocessing Routines

The preprocessing unit of CODE exists to supply the data required for the calculation of the coefficient matrices and load vectors for the problem to be solved. The definition of the various groups of data is made in subroutines called by PREP.

Routine RCON reads the value of the control parameters NNODE, NELEM, NMAT, and NPOINT. (See Fig. 4.4 for definitions of these parameters.) The allowable range of control parameters is limited by the storage capacity of array of the computer.

Routine RNODE reads data that allow the definition of the coordinates of all nodal points. The nodes are numbered consecutively from 1 through NNODE. The coordinates are stored in array Z and transmitted and nodes are generated with equal spacing in various sections of the grid.

Routine RELEM reads the data that allow the definition of all the elements in the finite element model. The elements are numbered consecutively from 1 through NELEM. The data that define element number I are introduced in the following way of arranging:

KIND(I) = kind of element (degree of polynomial in the shape function)

3 for cubic elements

$MAT(I), NCON(I) \ NINT(I) = \text{See Fig. 4.4.}$

$NODES(J,I) \ J = 1,2,...,(KIND(I)+1) = \text{nodal point numbers of nodes in element}$

These data are stored in the array in common block CELEM and transmitted as needed through this block. RELEM should read data records that allow the definition of the data described above. It is, of course, not desirable to require a separate data record to be read for each element being defined same. The following format of data records will introduce the complete specification of element data.

RECORD 1: NREC = number of element data records

RECORD 2: KINDI, MATI, NCONI, NINTI, NODEI, NUMBER

KINDI = value of KIND for elements to be generated

MATI = value of MAT for elements to be generated

NCONI = value of NCON for elements to be generated

NINTI = order of the integration rule

NODEI = nodal point number for first node of first element
to be generated

NUMBER = number of elements to be generated in sequence;
element number and nodal point numbers are incremented
as elements are generated.

The coding of RELEM must cause record 1 to be read, then a loop entered until record 2 is read and the appropriate number of elements generated.

Routine RBC reads data that defines the boundary condition at each end of the region. For each boundary condition, the data consists of a flag KBC and values VBC. The meaning of these variables is shown below. Note that $I = 1$ for the bottom end and $I = 2$ for the top end.

$KBC(J,I) = 1$ for boundary with the prescribed value, otherwise 0

$VBC(J,I) = \text{The prescribed values } (J = \text{odd; displacements, even; rotations})$

Routine RBC also reads the data point loads (if any). For each of the NPOINT loads, the nodal point number at which the loads acts, KPOINT, and the value of the load, POINT, are to be read. Further, the attached mass, damp, and spring at the top end are read in both lateral and rotational degrees if it is required. The capital letter R within the variables indicates the rotational degree.

The descriptions of routine RMAT, RCONS, and RPARA are given in Fig. 4.3 and the meaning of the variables is written below.

PROP(J,I) = material properties; I = element number

- J = 1; mass per unit length (riser mass + added mass + mud mass)
- 2; mass of mud per unit length
- 3; density of sea water
- 4; section area of riser
- 5; outside diameter of riser
- 6; the value of top tension
- 7; the value of bottom tension
- 8; coefficient of elasticity E
- 9; moment of inertia I

CONS(J,I) = input constants and velocities; I = element number

- J = 1; the velocity of internal fluid
- 2; current velocity (if necessary)
- 3; Cd
- 4; Ca
- need more constants for the analysis of vortex-induced vibration or vibration due to wave or current loading (See section 6.3 or 6.4)

DT = time interval for integration

TMAX = maximum time to be considered

TF = ending time for point force (if any)

ALPHA = parameter value for Newmark integration

DELTA = parameter value for Newmark integration

ER = error ratio for convergence = (new value- old value)/old value

4.4 Finite Element Calculation Routines

This unit consists of the calling routine PROC and the routines that calculates, assemble, modify for boundary conditions, and solve the system of matrix O.D.Es (ordinary differential equations). PROC contains the calls to the various components of the analysis procedure, which, in CODE, consists of successive calls to routines FORMGM, MODGMB, FORMGV, FMODGV, APLEFO, APLBCV, APLBCM, and, NEWMRK.

Routine FORMGM or FORMGV directs the calculation of element coefficient matrices or load vectors evaluated by routine ELEM or ELEMV and the accumulation into the global matrix or vector by ASSMBM or ASSMBV. These routines begin by zeroing the global matrices and load vector respectively. Then, element by element, the routines that calculate and assemble the element matrices (EM, EC, EK), and vectors (EFC), are called. The assembly routines (ASSMBM, ASSMBV) require the element and global matrices or vectors, and the nodal point numbers (equation numbers) for the nodes in the element. Since the node numbers for each element are stored consecutively in the array NODES, it suffices to transmit, through the calling sequence, only the location of the first node number of the element. Then, the contributions to the global matrices or vector from a single element are added to the accumulated contributions from other elements. Specially, in the accumulation of global matrices, full bandwidth technique is implemented to reduce the size of global matrices. In our problem, half-bandwidth technique is no longer adequate because some of our element matrices is not symmetric.

Routine MODGMB adds the concentrated mass, damp, and spring at the top end, by the accumulation of their contributions, to the corresponding location of global matrices and APLEFO adds the point loads, by the treatment of array KPOINT and NODES, to the selected position of global vector. In some type of boundary condition

(no movement at the top end from original position), the operation of MODGMB has no effect on the solutions because the rows and columns corresponding to top boundary are eliminated in subroutine APLBCM. Routine APLBCM and APLBCV eliminates unnecessary rows and columns due to fixed boundary values from global matrices and vector respectively.

Routine FMODGV modifies global vectors for initial prescribed values at top and bottom boundary. The modification for boundary conditions is made by calling routine DRCHLT representing the computer implementation of the essential boundary conditions, which may be found in several references related to finite element book.

Finally, routine NEWMRK performs Newmark integration, predictor-corrector scheme, and convergence in solution, and then prints results. This routine will be discussed in chapter 6 in details.

CHAPTER 5

SOLUTION OF THE FREE VIBRATION OF THE SYSTEM

For the investigation of the characteristics of a marine riser system with the inclusion of internal flow, it is generally required to estimate the system frequencies and mode shapes. However, it is not a simple task for the case of the system including highly nonlinear terms. Moreover, even the attempt to resolve the problem has not been made for the case of the system with deformation-dependent nonlinear terms like the present case. Thus, the system frequencies and mode shapes are obtained by solving the approximate form of governing equations derived in section 2.5.3 or 3.5. This serves our purpose to investigate the effect of internal flow on the system.

This chapter consists of three sections; first, semi-analytical solutions are obtained using series expansion for the mathematical model and second, numerical solutions are calculated adapting complex eigenvalue method for the matrix equilibrium equation. Finally, the algorithm for the computer program is introduced and verification is given by comparing the solutions from the first method to that from the second method. Further, the comparisons of semi-analytical solutions with results from other papers is presented.

5.1 Semi-Analytical Solution

The approximate governing equation for lateral vibration of a marine riser system with the internal flow and linear tension has been derived in section 2.5.3. The free vibration equation of the system can be derived simply by setting the forcing term in the right hand side of Eq. (2.68) to zero and dropping the subscript 1 of variable x indicating the lateral deformation. The resulting equation represents the free vibration of the system

in the lateral direction and is

$$m_i \ddot{x} + 2m_f V_1 \dot{x}' + m_f V_1^2 x'' + (EI x'')'' - wx' - T_e x'' = 0 \quad (5.1)$$

where the unknown variable x represents the displacement in lateral direction and prime denotes the derivative with respect to a vertical coordinate z .

Providing that pipe has an uniform section of constant EI and selecting T_e as the mean value of the tension force acting along the length of riser, Eq. (5.1) becomes

$$m_i \ddot{x} + 2m_f V_1 \dot{x}' + m_f V_1^2 x'' + EI x'''' - wx' - \bar{T}_e x'' = 0 \quad (5.2)$$

where $\bar{T}_e$ represents mean tension and is given as $TTB + wL/2$ or $TTR - wL/2$.

5.1.1 Quasi Mode Shape

Equation (5.2) differs from the usual vibration equations in that it contains a mixed derivative term with respect to time and space which is the coriolis force, the force required to rotate fluid elements with local pipe rotation. Its mixed derivative causes an asymmetric distortion of classical mode shapes and could lead to a flutterlike instability. Thus, Eq. (5.2) does not possess the classical normal modes. Its solution can not be separated simply into time and space components. For example, if a trial solution of the form

$$x(z, t) = \tilde{x}(z) \sin \omega t \quad (5.3)$$

is substituted into (5.2), it can be seen that the coriolis force term varies as $\cos \omega t$, while the remainder of the terms have a $\sin \omega t$ term. This suggests that solution should be written as

$$x(z, t) = a_1 \tilde{x}(z) \sin \omega t + a_2 \tilde{x}(z) \cos \omega t \quad (5.4)$$

The boundary conditions for a pinned-pinned span are given by

$$x(0, t) = x(L, t) = \partial^2 x(0, t) / \partial z^2 = \partial^2 x(L, t) / \partial z^2 = 0 \quad (5.5)$$

and those boundary conditions are satisfied by the set of sinusoidal mode shapes,

$$x(z) = \sin n\pi z / L, \quad n = 1, 2, 3, \dots \quad (5.6)$$

These mode shapes make the first, third, fourth, and sixth terms in Eq. (5.2) unaltered, but the mixed derivative in the coriolis force term generates spatially asymmetric terms for a symmetric mode shape ($n = 1, 3, 5, \dots$) and spatially symmetric terms for an asymmetric mode shape ($n = 2, 4, 6, \dots$). These considerations imply that the solution of (5.2) with pinned end conditions should be the sum of symmetric and asymmetric spatial modes with sine and cosine components (Housner, 1952; Blevins, 1990).

$$x_j(z, t) = \sum_{n=1,3,5,\dots} a_n \sin(n\pi z/L) \sin \omega_j t + \sum_{n=2,4,6,\dots} a_n \sin(n\pi z/L) \cos \omega_j t$$

$$j = 1, 2, 3, \dots \quad (5.7)$$

where ω_j is the natural frequency of the j -th vibration mode and the coefficient equation will be derived in next section.

Eq. (5.7) shows that the classical mode shape does not exist in our system as stated early. Moe (1988) has shown that there is a phase shift due to the internally flowing fluid. Chen has made a heuristic assumption that the time components have been dropped out of equation (5.7) at the end of his derivation and showed through his numerical example that the error due to the assumption was negligible as compared to other author's results from different approaches. According to his assumption, the 1st quasi mode shape can be formed by dropping the time components from (5.7) and given by

$$x_1(z) = \sum_{n=1,3,5,\dots} a_n \sin(n\pi z/L) + \sum_{n=2,4,6,\dots} a_n \sin(n\pi z/L) \quad (5.8)$$

where the coefficients was determined to the 1st natural frequency ω_1 .

In this paper, the quasi mode shape, which is not from Chen's assumption but by including time dependent component in Eq. (5.7), is implemented for the determination of system frequency.

5.1.2 Modified Eigenvalue Problem

In order to obtain the coefficient equations, let's begin with the substitution of trial solution (5.7) into Eq. (5.2). By this substitution, the coriolis force and the fifth term in (5.2) produce terms containing $\cos(n\pi z/L)$. These cosine terms can be expanded in a Fourier half-range series of sine functions

$$\cos(n\pi z/L) = \sum_{p=1,2,3,\dots} b_{np} \sin(p\pi z/L), \quad n = 1, 2, 3, \dots \quad (5.9)$$

where

$$b_{np} = \begin{cases} 0 & n + p = \text{even} \\ 4p / \{\pi(p^2 - n^2)\} & n + p = \text{odd} \end{cases} \quad (5.10)$$

This Fourier series converges relatively slowly over the span and not at all at the ends, but it does allow the spatial dependence to be factored out of the solution. With the substitution of the series expansion of cosine function, the terms in (5.2) can be grouped according to whether they contain $\sin \omega t$ or $\cos \omega t$ to give following equation:

$$\begin{aligned} & \sum_{n=1,3,5,\dots} a_n \left\{ -m_t \omega_j^2 - m_f V_1^2 (n\pi/L)^2 + \bar{T}_e (n\pi/L)^2 + EI (n\pi/L)^4 \right\} \sin(n\pi z/L) \sin(\omega_j t) \\ & + \sum_{n=1,3,5,\dots} (-8m_f V_1 \omega_j / L) \left\{ \sum_{p=2,4,6,\dots} (a_p p n) / (n^2 - p^2) \right\} \sin(n\pi z/L) \sin(\omega_j t) \\ & + \sum_{n=1,3,5,\dots} a_n \left\{ -w(n\pi/L) \left(\sum_{p=2,4,6,\dots} 4p / (\pi(p^2 - n^2)) \sin(p\pi z/L) \right) \right\} \sin(\omega_j t) \\ & + \sum_{n=2,4,6,\dots} a_n \left\{ -m_t \omega_j^2 - m_f V_1^2 (n\pi/L)^2 + \bar{T}_e (n\pi/L)^2 + EI (n\pi/L)^4 \right\} \sin(n\pi z/L) \cos(\omega_j t) \\ & + \sum_{n=2,4,6,\dots} (8m_f V_1 \omega_j / L) \left\{ \sum_{p=1,3,5,\dots} (a_p p n) / (n^2 - p^2) \right\} \sin(n\pi z/L) \cos(\omega_j t) \\ & + \sum_{n=2,4,6,\dots} a_n \left\{ -w(n\pi/L) \left(\sum_{p=1,3,5,\dots} 4p / (\pi(p^2 - n^2)) \sin(p\pi z/L) \right) \right\} \cos(\omega_j t) = 0 \quad (5.11) \end{aligned}$$

For a neutrally buoyant case ($w = 0$), Eq. (5.11) becomes

$$\begin{aligned}
& \sum_{n=1,3,5,\dots} a_n \left\{ -m_i \omega_j^2 - m_f V_1^2 (n\pi/L)^2 + \bar{T}_e (n\pi/L)^2 + EI (n\pi/L)^4 \right\} \sin(n\pi z/L) \sin(\omega_j t) \\
& + \sum_{n=1,3,5,\dots} (-8m_f V_1 \omega_j/L) \left\{ \sum_{p=2,4,6,\dots} (a_p p n)/(n^2 - p^2) \right\} \sin(n\pi z/L) \sin(\omega_j t) \\
& + \sum_{n=2,4,6,\dots} a_n \left\{ -m_i \omega_j^2 - m_f V_1^2 (n\pi/L)^2 + \bar{T}_e (n\pi/L)^2 + EI (n\pi/L)^4 \right\} \sin(n\pi z/L) \cos(\omega_j t) \\
& + \sum_{n=2,4,6,\dots} (8m_f V_1 \omega_j/L) \left\{ \sum_{p=1,3,5,\dots} (a_p p n)/(n^2 - p^2) \right\} \sin(n\pi z/L) \cos(\omega_j t) = 0 \quad (5.12)
\end{aligned}$$

The coefficients of each group are set to zero to give the following equations

$$a_n \left[EI (n\pi/L)^4 - m_f V_1^2 (n\pi/L)^2 + \bar{T}_e (n\pi/L)^2 - m_i \omega_j^2 \right] = (8m_f V_1 \omega_j/L) \sum_{p=2,4,6,\dots} a_p p n / (n^2 - p^2)$$

$$n = 1, 3, 5, \dots \quad (5.13)$$

$$a_n \left[EI (n\pi/L)^4 - m_f V_1^2 (n\pi/L)^2 + \bar{T}_e (n\pi/L)^2 - m_i \omega_j^2 \right] = -(8m_f V_1 \omega_j/L) \sum_{p=1,3,5,\dots} a_p p n / (n^2 - p^2)$$

$$n = 2, 4, 6, \dots \quad (5.14)$$

For a negatively or positively buoyant case ($w \neq 0$), the exact coefficient equations can not be obtained because the third and sixth terms in (5.11) have different running number in summation. It causes a slight shift in phase to remaining terms. However, its magnitude is small enough to ignore phase lag if a large number of summation is taken. Thus, the running number n and p can be switched in the summation of the third and sixth term of (5.11) with each other and then, Eq. (5.11) becomes

$$\begin{aligned}
& \sum_{n=1,3,5,\dots} a_n \left\{ -m_i \omega_j^2 - m_f V_1^2 (n\pi/L)^2 + \bar{T}_e (n\pi/L)^2 + EI (n\pi/L)^4 \right\} \sin(n\pi z/L) \sin(\omega_j t) \\
& + \sum_{n=1,3,5,\dots} (-8m_f V_1 \omega_j/L) \left\{ \sum_{p=2,4,6,\dots} (a_p p n)/(n^2 - p^2) \right\} \sin(n\pi z/L) \sin(\omega_j t) \\
& + \sum_{n=1,3,5,\dots} -4w/L \left(\sum_{p=2,4,6,\dots} a_p p n / (n^2 - p^2) \right) \sin(n\pi z/L) \sin(\omega_j t) \\
& + \sum_{n=2,4,6,\dots} a_n \left\{ -m_i \omega_j^2 - m_f V_1^2 (n\pi/L)^2 + \bar{T}_e (n\pi/L)^2 + EI (n\pi/L)^4 \right\} \sin(n\pi z/L) \cos(\omega_j t)
\end{aligned}$$

$$\begin{aligned}
& + \sum_{n=2,4,6,\dots} (8m_f V_i \omega_j / L) \left\{ \sum_{p=1,3,5,\dots} (a_p p n) / (n^2 - p^2) \right\} \sin(n\pi z / L) \cos(\omega_j t) \\
& + \sum_{n=2,4,6,\dots} -4w / L \left(\sum_{p=1,3,5,\dots} a_p p n / (n^2 - p^2) \right) \sin(n\pi z / L) \cos(\omega_j t) \quad (5.15)
\end{aligned}$$

Collecting Eq. (5.15) into groups according to $\sin \omega_j t$ or $\cos \omega_j t$ and setting the coefficient to zero, the following set of coefficient equations can be obtained:

$$\begin{aligned}
& a_n \left[EI(n\pi/L)^4 - m_f V_i^2 (n\pi/L)^2 + \bar{T}_e(n\pi/L) - m_t \omega_j^2 \right] \\
& = (8m_f V_i \omega_j + 4w) \sum_{p=2,4,6,\dots} a_p p n / (n^2 - p^2) \quad n = 1, 3, 5, \dots \quad (5.16)
\end{aligned}$$

$$\begin{aligned}
& a_n \left[EI(n\pi/L)^4 - m_f V_i^2 (n\pi/L)^2 + \bar{T}_e(n\pi/L) - m_t \omega_j^2 \right] \\
& = (-8m_f V_i \omega_j + 4w) \sum_{p=1,3,5,\dots} a_p p n / (n^2 - p^2) \quad n = 2, 4, 6, \dots \quad (5.17)
\end{aligned}$$

These equations can be put in matrix form:

$$\{[K_{np}] - \omega_j [C_{np}] - \omega_j^2 m_t [I]\} \{a\} = 0 \quad (5.18)$$

where,

$$K_{np} = \begin{cases} EI(n\pi/L)^4 + \bar{T}_e(n\pi/L)^2 - m_f V_i^2 (n\pi/L)^2, & n = p \\ (-4w/L) \{pn / (n^2 - p^2)\}, & n = \text{odd}, \quad p = \text{even} \\ (-4w/L) \{pn / (n^2 - p^2)\}, & n = \text{even}, \quad p = \text{odd} \\ 0, & n \neq p, \quad n + p = \text{even} \end{cases} \quad (5.19)$$

$$C_{np} = \begin{cases} 0, & n = p \\ (8m_f V_i / L) \{pn / (n^2 - p^2)\}, & n = \text{odd}, \quad p = \text{even} \\ (-8m_f V_i / L) \{pn / (n^2 - p^2)\}, & n = \text{even}, \quad p = \text{odd} \\ 0, & n \neq p, \quad n + p = \text{even} \end{cases} \quad (5.20)$$

$$\{a\} = \{a_1, a_2, a_3, \dots\}^T$$

$[I]$ = the identity matrix with value of one on the diagonal and all other equal to zero.

Nontrivial solution of Eq. (5.18) are sought by setting the determinant of the coefficient matrix to zero:

$$|[K] - \omega [C] - \omega^2 m_t [I]| = 0 \quad (5.21)$$

Because the system has an infinite number of natural modes, we have to consider only the first few modes practically. Blevins (1990) included only the first two modes in his appropriate analysis, then $a_3, a_4, a_5, \dots$ are set equal to zero. In this paper, the more number of natural modes will be included in practical analysis.

5.2 MDOF System with Coriolis Matrix

Since the semi-analytical method included the assumption in derivation and the simplicity in a practical sense as shown in the previous section, the error resulting from the assumption and the simplicity should be checked. The conventional way for determining the system frequency is to construct eigenvalue problem using the matrix equilibrium equation of free vibration system which is already constructed in section 3.5 as given the following equation:

$$[\tilde{M}]\{\ddot{x}(t)\} + [\tilde{C}_{ij}]\{\dot{x}(t)\} + [\tilde{T}_{ij} - \tilde{C}_{ej} + \tilde{K}_{ij}]\{x(t)\} = 0 \quad (5.22)$$

Rewriting (5.22) in a usual simple form, we get

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{0} \quad (5.23)$$

In above equation, $\mathbf{C}$ is not a usual damping matrix but a coriolis matrix which is skew-symmetric. As discussed earlier, $\mathbf{C}\dot{\mathbf{U}}$ acts like a dynamic coupling force not a damping force. Thus, it can not be allowed to construct eigenvalue problem simply using Duncan's method (1956) because the imaginary part of eigenvalue, which indicates a damped frequency, is always zero. The complex value with zero imaginary should be implemented and then, the imaginary part has to be checked whether it is equal to zero. This is verified through the numerical example given in 5.3.2.

For the construction of eigenvalue problem using Duncan's method, we combine the identity

$$\mathbf{M}\dot{\mathbf{U}} - \mathbf{M}\dot{\mathbf{U}} = 0 \quad (5.24)$$

with Eq. (5.23) to get

$$\begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}} \\ \dot{\mathbf{U}} \end{Bmatrix} + \begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}} \\ \mathbf{U} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \quad (5.25)$$

Equation (5.25) can now be rewritten as

$$\mathcal{M}\dot{\mathcal{U}} + \mathcal{K}\mathcal{U} = 0 \quad (5.26)$$

where

$$\mathcal{M} = \begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix}$$

$$\mathcal{K} = \begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix}$$

$$\mathcal{U} = \begin{Bmatrix} \dot{\mathbf{U}} \\ \mathbf{U} \end{Bmatrix} \quad \text{and} \quad \dot{\mathcal{U}} = \begin{Bmatrix} \ddot{\mathbf{U}} \\ \dot{\mathbf{U}} \end{Bmatrix} \quad (5.27)$$

Now letting

$$\mathcal{U} = \mathbf{V}e^{pt} \quad (5.28)$$

where p is complex. Eq. (5.26) can be transformed into

$$(p\mathcal{M} + \mathcal{K})\mathbf{V} = 0 \quad (5.29)$$

This last equation is of a standard form for which eigenvalue computer programs are available. The disadvantage we have for this simplification is that the sizes of all matrices are doubled.

5.3 Numerical Evaluation of System Frequency

5.3.1 Algorithm for Computer Program

In the previous section, eigenvalue problem with the form of (5.18) and (5.29) has been derived. Eq. (5.29) is of a standard form for which the algorithms to obtain the required eigenvalues and eigenvectors are available, on the other hand, (5.18) is not in the standard form. Thus, it is required to implement a slightly different algorithm based on one for the standard form of eigenvalue problem. As recognized in the algorithm summarized in Fig. 5.1, the standard form of eigenvalue problem has to be resolved for every iteration step.

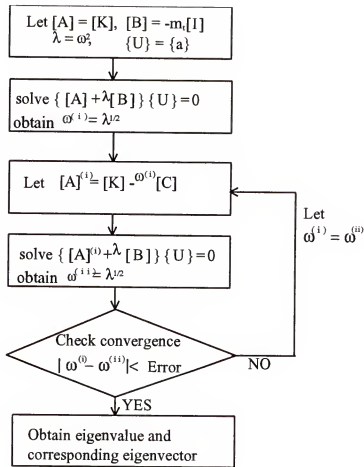


Figure 5.1: Algorithm for Solving Non-Standard Eigenvalue Problem

In this paper, the inverse iteration algorithm with shifting value is chosen to solve the given eigensystem. The technique of inverse iteration is very effectively used to calculate an eigenvector, and at the same time the corresponding eigenvalue can also be evaluated. Inverse iteration, like a usual vector iteration method, can be employed even if some diagonal elements of mass matrix has zero value as shown in Eq. (5.27). Further, the shift of eigenvalue improves the convergence rate in the vector iteration and accelerates the calculation of the eigenvectors. Those techniques are utilized together in this paper and the computer program, of which algorithms are referenced from the text of Bathe and Wilson (1976), is developed in this study. In addition to the subroutines in computer program, subroutine FORMA which constructs the coefficient matrices in (5.18) and first step of Fig. 5.1, and FORMB which forms the modified stiffness matrix $[A]^{(i)}$ for every iteration step as shown in Fig. 5.1 are built in computer program. While FORMA and FORMB are necessary routines for the calculation of the eigensystem by first method (semi-analytical method), FORMA is only routine for preparing coefficient matrices M and K in Eq. (5.29), whose elements are filled with the resulting coefficients from the matrix equilibrium equation (Eq. (5.22) and (5.23)) constructed by finite element approximation (see section 3.5).

5.3.2 Comparison of Results

For the verification of the solutions from semi-analytical method, the comparisons are made in two different ways. The first is to calculate the system frequencies of the system with the exclusion of internal flow using semi-analytical method and then compare those with the results from previous investigators. This comparison provides the validity of the representation of solution into the sum of two components as shown in Eq. (5.7) but it does not supply enough evidence for the system including the internal flow. Thus, another one in section 5.2 is implemented to compare with the results from the semi-analytical method. This comparison gives enough evidence even for the system including

internal flow. The comparison with Chen's results (1992) may be also given. However, in the expansion of cosine term into Fourier series, he followed the wrong use of Fourier coefficients which was also found in Housner's paper (1952). The wrong expansion of the coriolis force term may lead to the error in the calculation of the system frequency with internal flow.

The example data, which is for a typical drilling riser system for use in the northern North sea and repeated from Chen (1992), is given in Fig. 5.2. Using those data, the first comparison is given in Table 5.1 which presents the comparison with the natural frequencies of the 1st to 10th modes obtained from semi-analytical method, Dareing (1976), Spanos (1980), Kim (1988), and Chen (1992). The discrepancy between those methods including semi-analytical method is negligible, that is, the representation of solution such as Eq. (5.7) is acceptable.

Outside diameter	$D = 2.0$ ft, with 5/8" wall thickness
Constant of elasticity	$E = 30 \times 10^6$ psi
Sectional moment of inertia	$I = 0.1518$ ft ⁴
Riser length	$L = 500$ ft
Riser mass	$m = 20.86$ slug/ft (includes mass of drilling mud and sea water)
Bottom tension	TTB = 286,000 lb
Effective weight per unit length	$w = 264.68$ lb/ft
Mean tension	$\bar{T}_e = 326,830.5$ lb
Density of drilling mud	$\rho_m = 85$ lb/ft ³
Density of sea water	$\rho_w = 64.8$ lb/ft ³

Figure 5.2: The Design Properties and Data for a Drilling Riser System for Use in the Northern North Sea

Table 5.1: Comparison of System Frequencies of the 1st to 10 th Modes for a Drilling Riser System with No Internal Flow. (System frequency = ω_i (rad/sec), $V_1 = 0$ m/sec)

Mode i	Semi-Analytical Method	Chen	Dareing	Kim	Spanos
1	0.81813	0.81585	0.81498	0.81484	0.831
2	1.80520	1.80430	1.80362	1.80390	1.831
3	3.08798	3.08585	3.08762	3.08328	3.123
4	4.73703	4.73268	4.73748	4.73328	4.778
5	6.78865	6.78123	6.78901	6.78934	6.832
6	9.26100	9.24969			9.301
7	12.1634	12.14743			12.193
8	15.5008	15.47942			15.506
9	19.2760	19.24850			19.232
10	23.4907	23.45631			23.352

For the riser system with internal flow of 6 m/sec, the comparisons of the 1st to 10th modal frequencies calculated using both semi-analytical method and numerical method described in section 5.2 are given in Table 5.2. The system frequencies obtained using second method are complex values which consist of real and imaginary part. The imaginary part represents damping frequency. As shown in Table 5.2, the imaginary parts have almost zero values, that is, the mixed derivative term in governing equation or the coriolis skew-symmetric term in matrix equilibrium equation does not have the same effect as that of viscous damping force although it involves the time derivatives. In other words, the mixed derivative term or the coriolis matrix indicates the dynamic coupling

Table 5.2: Comparison of System Frequencies of the 1st to 10 th Modes for a Drilling Riser System with Internal Flow. (System frequency = ω_i (rad/sec), $V_I = 6$ m/sec)

Mode i	Semi-Analytical Method	Numerical method described in section 5.2	
		Real value	Imaginary value
1	0.81701	0.81417	0.00231
2	1.80326	1.80030	0.00194
3	3.08550	3.06014	0.00102
4	4.73421	4.73527	0.00032
5	6.78562	6.78776	0.00003
6	9.25782	9.26105	0.000007
7	12.1601	12.1916	0.000001
8	15.4975	15.4908	0.000000
9	19.2727	19.2735	0.000000
10	23.4889	23.4909	0.000000

force as discussed earlier. Further, it can be recognized from the table that the discrepancy in the system frequencies resulting from the two different methods is negligible. Thus, both two methods developed in this chapter are acceptable for the estimation of the system frequency regardless of the existence of internal flowing fluid.

Of two methods, semi-analytical method is more convenient than the other because there is no need to construct the coefficient matrices through the finite element approximation. Thus, semi-analytical method will be adapted for the investigation of the effect of internal flow on system frequency.

CHAPTER 6 FORCED VIBRATION OF THE SYSTEM

This chapter deals with the forced vibration of the system derived in chapter 3 and its dynamic behaviors. The system considered here consists of two; first, the system simply responding to the general environmental loads such as wave or current and second, the system responding to the vortex loads generated in the transverse direction to inline current, i.e., vortex-induced vibration. Newmark method is used for time step integration and predictor-corrector scheme is implemented as an iteration algorithm for the nonlinear terms. Using the techniques mentioned above, vortex-induced vibration due to current and inline vibrations due to current or wave are obtained. And also, the application of top boundary condition to the system matrices or vectors is discussed and its effect to the responses of a riser is inspected.

6.1 Newmark Method for Time Step Integration

In chapter 3, we derived the equations of motion for a riser, which can be written in the following general form

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{R} \quad (6.1)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are coefficient matrices; $\mathbf{R}$ is external load vector; and $\mathbf{U}$, $\dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$ are the displacement, velocity, and acceleration vectors of the finite element assemblage. However, it should be noted that although Eq. (6.1) seems to be linear in its representation, it is actually nonlinear because the equation is written in the abridged form of Eq. (3.51), (3.61), (3.62), and (3.63). Thus, the coefficient matrices or vectors in (6.1) are also determined by comparing (6.1) with (3.51), (3.61), and so on.

Previous studies (Bathe and Wilson, 1976) have shown that Newmark method is the most convenient and economical one to employ a time step integration because larger time steps can be used and still obtain accurate values for the response. Its stability analysis has been done for linear systems. They do not, however, carry over to nonlinear case up to now. Unfortunately, a relatively large time step may lead to the divergence in solution. Thus, it is generally suggested to follow the Newmark's algorithm developed for linear case and apply simple iteration outside the loop of its flow or another scheme for iteration (See next section.) to remove the possibility of unbounded solution.

The Newmark integration scheme is an extension of the linear acceleration method. The following assumptions are used:

$$\dot{\mathbf{U}}_{t+\Delta t} = \dot{\mathbf{U}}_t + \left[(1 - \delta) \ddot{\mathbf{U}}_t + \delta \ddot{\mathbf{U}}_{t+\Delta t} \right] \Delta t \quad (6.2)$$

$$\mathbf{U}_{t+\Delta t} = \mathbf{U}_t + \dot{\mathbf{U}}_t \Delta t + \left[(1/2 - \alpha) \ddot{\mathbf{U}}_t + \alpha \ddot{\mathbf{U}}_{t+\Delta t} \right] \Delta t^2 \quad (6.3)$$

where α and δ are parameters that can be determined to obtain integration accuracy and stability. When $\delta = \frac{1}{2}$ and $\alpha = \frac{1}{6}$, relations (6.2) and (6.3) correspond to the linear acceleration method. Newmark originally proposed, as an unconditionally stable scheme, the constant average acceleration method, in which case $\delta = \frac{1}{2}$ and $\alpha = \frac{1}{4}$ (See Fig. 6.1).

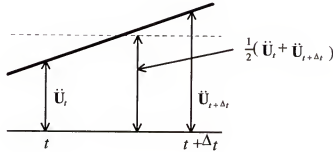


Figure 6.1: Newmark's Constant Average Acceleration Scheme

In addition to (6.1) and (6.2), for solution of the displacements, velocities, and accelerations at time $t + \Delta t$, the equilibrium equations (6.1) at time $t + \Delta t$ are also considered:

$$\mathbf{M} \ddot{\mathbf{U}}_{t+\Delta t} + \mathbf{C} \dot{\mathbf{U}}_{t+\Delta t} + \mathbf{K} \mathbf{U}_{t+\Delta t} = \mathbf{R}_{t+\Delta t} \quad (6.4)$$

Solving from (6.3) for $\ddot{\mathbf{U}}_{t+\Delta t}$ in terms of $\mathbf{U}_{t+\Delta t}$, and then substituting for $\ddot{\mathbf{U}}_{t+\Delta t}$ into (6.2), we obtain equations for $\ddot{\mathbf{U}}_{t+\Delta t}$ and $\dot{\mathbf{U}}_{t+\Delta t}$, each in terms of the unknown displacements $\mathbf{U}_{t+\Delta t}$ only. These two relations for $\dot{\mathbf{U}}_{t+\Delta t}$ and $\ddot{\mathbf{U}}_{t+\Delta t}$ are substituted into (6.4) to solve for $\mathbf{U}_{t+\Delta t}$, after which, using (6.2) and (6.3), $\ddot{\mathbf{U}}_{t+\Delta t}$ and $\dot{\mathbf{U}}_{t+\Delta t}$ can also be calculated. The complete algorithm using Newmark integration scheme is given in Figure 6.2.

A. Initial Calculations:

1. Form $\mathbf{K}$, $\mathbf{M}$, and $\mathbf{C}$.
 2. Initialize $\mathbf{U}_0$, $\dot{\mathbf{U}}_0$, and $\ddot{\mathbf{U}}_0$.
 3. Select time step size Δt , parameters α and δ , and calculate integration constants:

$$\delta \geq 0.50, \quad \alpha \geq 0.25(0.5 + \delta)^2,$$

$$a_0 = 1/(\alpha \Delta t^2), \quad a_1 = \delta/(\alpha \Delta t), \quad a_2 = 1/(\alpha \Delta t), \quad a_3 = 0.5\alpha - 1,$$

$$a_4 = \delta/\alpha - 1, \quad a_5 = 0.5\Delta t(\delta/\alpha - 2), \quad a_6 = \Delta t(1 - \delta), \quad a_7 = \delta \Delta t$$
 4. Form effective stiffness matrix $\hat{\mathbf{K}}$: $\hat{\mathbf{K}} = \mathbf{K} + a_0 \mathbf{M} + a_1 \mathbf{C}$.
-

B. For Each Time Step:

1. Calculate effective loads at time $t + \Delta t$:

$$\hat{\mathbf{R}}_{t+\Delta t} = \mathbf{R}_{t+\Delta t} + \mathbf{M}(a_0 \mathbf{U}_t + a_2 \dot{\mathbf{U}}_t + a_3 \ddot{\mathbf{U}}_t) + \mathbf{C}(a_1 \mathbf{U}_t + a_4 \dot{\mathbf{U}}_t + a_5 \ddot{\mathbf{U}}_t)$$
 2. Solve for displacements at time $t + \Delta t$:

$$\hat{\mathbf{K}} \mathbf{U}_{t+\Delta t} = \hat{\mathbf{R}}_{t+\Delta t}$$
 3. Calculate accelerations and velocities at time $t + \Delta t$:

$$\ddot{\mathbf{U}}_{t+\Delta t} = a_0 (\mathbf{U}_{t+\Delta t} - \mathbf{U}_t) - a_2 \dot{\mathbf{U}}_t - a_3 \ddot{\mathbf{U}}_t$$

$$\dot{\mathbf{U}}_{t+\Delta t} = \dot{\mathbf{U}}_t + a_6 \ddot{\mathbf{U}}_t + a_7 \ddot{\mathbf{U}}_{t+\Delta t}$$
-

Figure 6.2: Algorithm for Step-by-Step Solution Using Newmark Integration Method.

6.2 Predictor-Corrector Scheme

The utilization of the kinematic constraint on unit tangent vector makes it feasible to represent the vertical degrees of freedom in terms of the lateral ones as stated in chapter 2. This does lead to the reduction of the degrees of freedom per element from 12 to 8 and remove the possibility of the divergence in solutions due to the iteration of highly nonlinear terms in the vertical degrees of freedom. Moreover, it can be implemented as an another algorithm scheme for the iteration due to nonlinear terms. For the development of an algorithm, so called predictor-corrector scheme, the derivation of the incremental form of the extensibility condition is necessary because the equations of equilibrium for a finite element system in motion to be solved is of the incremental form.

Using the kinematic constraint on unit tangent vector of riser, i.e. $\hat{t} \cdot \hat{t} = \mathbf{r}' \cdot \mathbf{r}' = 1$, we have the following relationship:

$$x_1'^2 + x_2'^2 + (x_3' + 1/(1 + \varepsilon_t))^2 = 1 \quad (6.5)$$

Applying the incremental operator Δ on above equation, we obtain

$$\Delta x_3' = - (x_1' \Delta x_1' + x_2' \Delta x_2') (x_3' + 1/(1 + \varepsilon_t))^{-1} + \Delta \varepsilon_t (1 + \varepsilon_t)^{-2} \quad (6.6)$$

$$\text{Also, we have } \mathbf{T}_e = \mathbf{F} \cdot \hat{t} = \mathbf{F} \cdot \mathbf{r}' = F_1 x_1' + F_2 x_2' + F_3 (x_3' + 1/(1 + \varepsilon_t)) \quad (6.7)$$

Applying the incremental operator on (6.7), we get

$$\begin{aligned} \Delta T_e = & \Delta F_1 x_1' + \Delta F_2 x_2' + \Delta F_3 (x_3' + 1/(1 + \varepsilon_t)) + F_1 \Delta x_1' + F_2 \Delta x_2' \\ & + F_3 \Delta x_3' - F_3 \Delta \varepsilon_t (1 + \varepsilon_t)^{-2} \end{aligned} \quad (6.8)$$

From Eq. (2.16) and (2.17), we have

$$\mathbf{T}_e = E A \varepsilon_t + \rho_w A_o g (S_w - S - x_3) - \rho_m A_i g (S_m - S - x_3) \quad (6.9)$$

Applying Δ again on (6.9), we obtain

$$\Delta T_e = E A \Delta \varepsilon_t - \Delta x_3 g (\rho_w A_o - \rho_m A_i) \quad (6.10)$$

Eliminating ΔT_e from (6.9) and (6.10), we compute $\Delta \epsilon_t$

$$\Delta \epsilon_t = \frac{\Delta F_1 x'_1 + \Delta F_2 x'_2 + \Delta F_3 (x'_3 + 1/(1 + \epsilon_t)) + F_1 \Delta x'_1 + F_2 \Delta x'_2 + F_3 \Delta x'_3 + \Delta x_3 g(\rho_w A_o - \rho_m A_i)}{EA + F_3 (1 + \epsilon_t)^{-2}} \quad (6.11)$$

Substituting (6.11) into (6.6) and solving for $\Delta x'_3$, we obtain

$$\begin{aligned} \Delta x'_3 = & \left(1 + \frac{F_3}{EA (1 + \epsilon_t)^2} \right) - \frac{x'_1 \Delta x'_1 + x'_2 \Delta x'_2}{x'_3 + 1/(1 + \epsilon_t)} + \frac{1}{EA (1 + \epsilon_t)^2 + F_3} \left\{ \Delta F_1 x'_1 + \Delta F_2 x'_2 \right. \\ & \left. + \Delta F_3 (x'_3 + 1/(1 + \epsilon_t)) + F_1 \Delta x'_1 + F_2 \Delta x'_2 + \Delta x_3 g(\rho_w A_o - \rho_m A_i) \right\} \end{aligned} \quad (6.12)$$

In addition to (6.12), we also have

$$\begin{aligned} \Delta x_3 &= \int_0^s \Delta x'_3 ds_1 \\ x'_3 &= (1 - x_1'^2 - x_2'^2)^{1/2} - 1/(1 + \epsilon_t) \\ x_3 &= x_3(0) + \int_0^s x'_3 ds_1 \end{aligned} \quad (6.13)$$

As stated earlier, the vertical degrees of freedom can be computed in terms of the lateral ones. For each load increment the kinematic relations, (6.12) and (6.13), are used as predictors to remove the vertical degrees of freedom from global matrix equation of equilibrium. They are used again as correctors, after the solution of the reduced system of equations has been achieved. During prediction phase $x_3, x'_3, \Delta x_3, \Delta x'_3$ are computed using (6.12) and (6.13). In addition, during the prediction phase equation (3.51) are used to compute $x_1, x'_1, \Delta x_1, \Delta x'_1, x_2, x'_2, \Delta x_2, \Delta x'_2$. During the correction phase $x_3, x'_3, \Delta x_3, \Delta x'_3$ are recomputed using (6.12)-(6.13) and all deformation dependent matrices, the equivalent nodal forces, the boundary conditions and the lengths of elements are corrected. Further $x_1, x'_1, \Delta x_1, \Delta x'_1, x_2, x'_2, \Delta x_2, \Delta x'_2$ are recomputed using (3.51). This predictor-corrector scheme is repeated until convergence is achieved for each load increment. The algorithm is summarized in Figure 6.3.

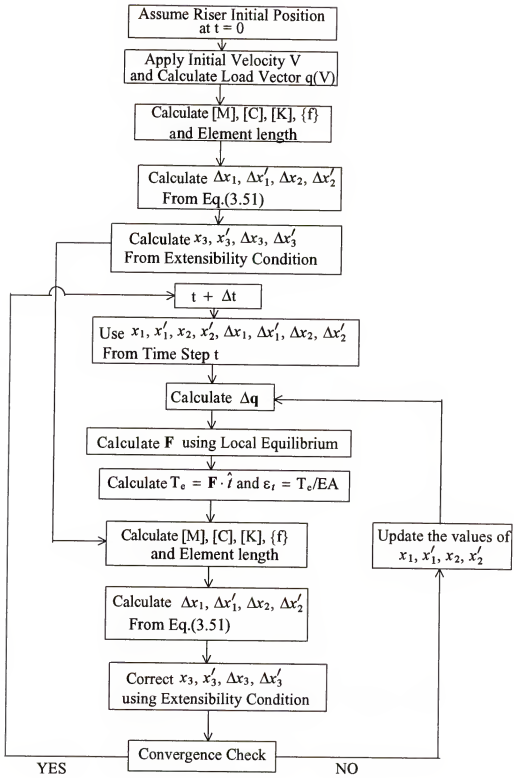


Figure 6.3: Solution Algorithm Employing Predictor-Corrector Scheme

6.3 Vortex-Induced Vibration Due to Current

In section 3.5, the matrix equation of equilibrium for current-vortex model (Eqs. (3.61)-(3.63)) is derived. As discussed in the section, Eqs. (3.61) and (3.62) represent the self-excited oscillator equations, while (3.63) presents inline vibration due to current. In those equations, the system matrix $\hat{M}_{ij}$ and $\tilde{M}_{ij}$ represent the system mass, $\hat{C}_{ij}$ and $\tilde{C}_{ij}$ coriolis force terms, $\hat{C}_{fij}$ and $\bar{C}_{ij}$ fluid damping terms, $\hat{T}_{ij}$ and $\tilde{T}_{ij}$ stiffness terms due to effective tension, $\hat{C}_{eij}$ and $\tilde{C}_{eij}$ stiffness terms due to centrifugal force, $\hat{K}_{ij}$ and $\tilde{K}_{ij}$ stiffness terms due to bending of pipe, $\hat{F}_{ij}$ and $\bar{F}_{ij}$ self-excited forcing terms due to the vortex shedding generated by inline current, and $\bar{M}_{ij}$ and $\bar{K}_{ij}$ represent mass and stiffness term of fluid oscillator respectively. And also, the system vector f_j represents the nonlinear drag damping due to current and $\bar{f}_j$ represents the nonlinear forcing term in fluid oscillator equation. The matrices representing coriolis force term, which cause the distortion of the response of a riser, are skew symmetric, while the others are symmetric.

From the inspection of coefficient matrices and vectors in (3.64), it may be recognized that the drag force in inline vibration equation and the fluid force in fluid oscillator equation have nonlinear term. In other words, there is no highly nonlinear terms in the equation to be solved, so that simple iteration can be applied for the convergence of solution instead of predictor-corrector scheme. Thus, the application of Newmark method combined with the simple iteration is enough to solve the system described in section 3.5. Using the method mentioned above, a finite element code, which is called CODEV, is developed. Overall structure of the code is the same as that of CODE given in Fig. 4.2 but some of the routines should be modified with slightly different arrays and variables because of the different coefficient matrices and vectors of system as shown in (3.61)-(3.64). For instance, in routine ELEM, ELEMV, GETCON, new array and variables such as a hidden flow variable w should be defined throughout CODEV. And also,

variable $CONS(J,I)$ in RCON, one of input routine, need more constants as written below.

$CONS(J,I)$ = input constants and velocities; I = element number

$J = 1$; the velocity of internal fluid

2 ; current velocity at top

3 ; current velocity at bottom

4 ; C_d

5 ; a_4

6 ; $p(x_3)$

7 ; a_4'

8 ; a_1'

9 ; ω_s

10 ; a_2'

11 ; C_a

$J = 5 - 10$; See section 2.5.4.

Further, routine NEWMRK should be coded to perform time step integration and simple iteration as discussed above. Figure 6.4 shows the flow charts of components of subroutine NEWMRK which completes Fig. 4.2. Routine NEWMRK performs Newmark time step integration and calls other subroutines within the routine. Routine FORMGV,

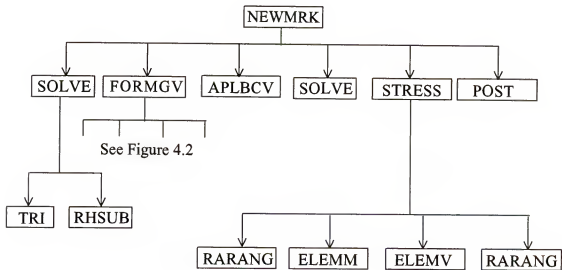


Figure 6.4: Flow Charts of Components of Subroutine NEWMRK in CODEV

APLBCV, and SOLVE just belong to the iteration phase, while the entire routine are operated for every time step. Routine STRESS calculates the internal forces and moments, and RARANG rearranges response vectors to include or exclude the fixed values at boundary.

For the verification of CODEV, the numerical results are inspected through the physical interpretation with the generally expected trends. The same material properties and constants of riser, which have been already applied in section 5.3.2, are selected again in this section. The first concern is that the deflected shape of riser should be symmetrical when the riser system, with the exclusion of internal tension and internal flow, is subject to a uniform current in the water column. Table 6.1 shows the responses of riser at certain time in a uniform current. Nodal displacements and rotations are shown to be exactly symmetrical or skew-symmetrical with respect to middle node 26 (typed " * " in front of node number), that is, the deflected shapes are exactly symmetrical when there are no internal tension and flow. Next, including internal tension and internal flow, the response of riser is examined. Figure 6.5 presents maximum transverse displacement versus current velocity. Since the transverse vibrations due to current are controlled by the oscillating vortex generated around the riser, they must have resonant bands near the critical current velocity in which the vortex shedding frequency accords with the system frequency. For instance, the 1st system frequency is already obtained from Table 5.2. The vortex shedding frequency can be also calculated from Eq. (2.72). Taking Strouhal number S as 0.20, the critical velocity can be obtained from the following equation:

$$\begin{aligned}\omega_s &= 2\pi S u_c / D_o = \omega_1 \\ \omega_s &= 2\pi(0.20) u_c / \{(2.0)(0.3048)\} = 0.81701 \\ u_c &= 0.40 \text{ m/sec} \quad (6.14)\end{aligned}$$

Table 6.1: Nodal Displacements and Rotations Due to Uniform Current in Transverse and Inline Direction (Time = 5 sec, Top Tension = 0, w = 0, Internal Velocity = 0)

Node #	Transverse Response		Inline Response	
	Displacement (m)	Rotation (rad)	Displacement (m)	Rotation (rad)
1	.00000D+00	.83193D-05	.00000D+00	.78644D-02
2	.66594D-01	.84558D-05	.23945D-01	.78390D-02
3	.13422D+00	.84810D-05	.47737D-01	.77651D-02
4	.20041D+00	.81095D-05	.71236D-01	.76467D-02
5	.26290D+00	.75911D-05	.94310D-01	.74872D-02
6	.32099D+00	.70087D-05	.11684D+00	.72902D-02
7	.37484D+00	.65810D-05	.13872D+00	.70590D-02
8	.42619D+00	.63184D-05	.15984D+00	.67973D-02
9	.47514D+00	.60141D-05	.18013D+00	.65091D-02
10	.52311D+00	.61228D-05	.19950D+00	.61985D-02
:	:	:	:	:
:	:	:	:	:
18	.77795D+00	.29004D-05	.31556D+00	.32081D-02
19	.80231D+00	.32003D-05	.32472D+00	.28022D-02
20	.82770D+00	.31212D-05	.33264D+00	.23961D-02
21	.85125D+00	.27309D-05	.33933D+00	.19911D-02
22	.87054D+00	.21307D-05	.34478D+00	.15881D-02
23	.88521D+00	.15961D-05	.34901D+00	.11874D-02
24	.89656D+00	.12147D-05	.35203D+00	.78929D-03
25	.90425D+00	.69518D-06	.35383D+00	.39377D-03
*26	.90711D+00	.14229D-16	.35443D+00	.11152D-14
27	.90425D+00	-.69518D-06	.35383D+00	-.39377D-03
28	.89656D+00	-.12147D-05	.35203D+00	-.78929D-03
29	.88521D+00	-.15961D-05	.34901D+00	-.11874D-02
30	.87054D+00	-.21307D-05	.34478D+00	-.15881D-02
31	.85125D+00	-.27309D-05	.33933D+00	-.19911D-02
32	.82770D+00	-.31212D-05	.33264D+00	-.23961D-02
33	.80231D+00	-.32003D-05	.32472D+00	-.28022D-02
34	.77795D+00	-.29004D-05	.31556D+00	-.32081D-02
:	:	:	:	:
:	:	:	:	:
42	.52311D+00	-.61228D-05	.19950D+00	-.61985D-02
43	.47514D+00	-.60141D-05	.18013D+00	-.65091D-02
44	.42619D+00	-.63184D-05	.15984D+00	-.67973D-02
45	.37484D+00	-.65810D-05	.13872D+00	-.70590D-02
46	.32099D+00	-.70087D-05	.11684D+00	-.72902D-02
47	.26290D+00	-.75911D-05	.94310D-01	-.74872D-02
48	.20041D+00	-.81095D-05	.71236D-01	-.76467D-02
49	.13422D+00	-.84810D-05	.47737D-01	-.77651D-02
50	.66594D-01	-.84558D-05	.23945D-01	-.78390D-02
51	.00000D+00	-.83193D-05	.00000D+00	-.78644D-02

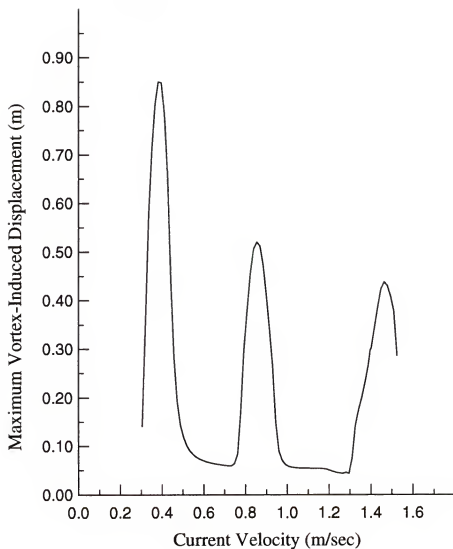


Figure 6.5: Maximum Vortex-Induced Displacement in Transverse Direction According to the Variation of Uniform Inline Current. The System Includes Internal Tension and Internal Flow. (Internal Velocity = 6.0 m/sec)

This critical velocity is consistent with the velocity in which the first peak takes place as shown in Fig. 6.5. In the same manner, the critical velocities for second and third peak in Fig. 6.5 are compared with the values calculated using Eq. (6.14) and found to be consistent with each other. While the loading due to vortex shedding in the transverse direction is oscillating well to induce harmonic riser motion, the constant load due to current force in inline direction causes a steady static response. This inline displacement simply becomes larger when the current velocity increases. Figure 6.6 presents maximum inline displacement due to uniform current which shows the trend as stated above. Figure 6.7 plots the time history of the transverse displacement at the middle point of the riser. In this figure, three current velocities, over, at and below first critical velocity are chosen to demonstrate the magnification in amplitude of oscillating displacement resulting from the current velocity that generates resonant vortices. The middle figure corresponds to the case of resonance. As discussed already, all three figures represent harmonically oscillating ones due to the vortex shedding. These harmonic responses due to vortex shedding are calculated by forcing riser with an imposed initial force. The vortex driven oscillation usually reaches to a steady state in about 10 - 15 cycles for top-hinged case. Also, the slight modulation in amplitude can be detected from the figure. This modulation results from a third power term in the hidden flow variable w on the right side of fluid oscillator equation (2.72). This can be proved from the fact that this modulation would disappear if the third power term is excluded from the fluid oscillator equation. Figure 6.8 plots the time history of inline displacement at the middle of riser. As expected, the displacement in the current direction approaches to a steady static state after a few cyclic fluctuations. Figure 6.9 plots the time-dependent trajectories of displacement vector at the middle of the riser. The first trajectory represents the trajectory for a current velocity below first critical velocity, the second is for the first critical velocity, and the third is for the case above the first critical velocity. As discussed in Fig. 6.5 through 6.8, similar differences

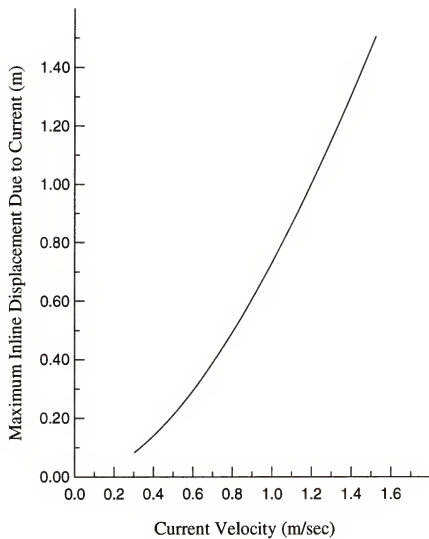


Figure 6.6: Maximum Inline Displacement in Current Direction According to the Variation of Uniform Inline Current. The System Includes Internal Tension and Internal Flow. (Internal Flow Velocity = 6.0 m/sec)

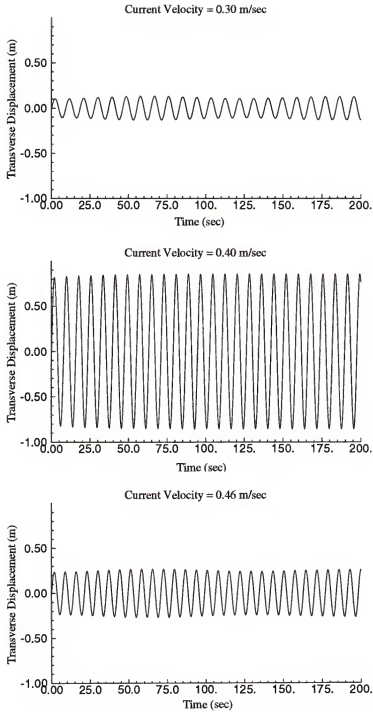


Figure 6.7: Transverse Displacement Due to Inline Current at the Middle Node of Riser According to Change of Time. The System Includes Internal Tension and Internal Flow of 6.0 m/sec.

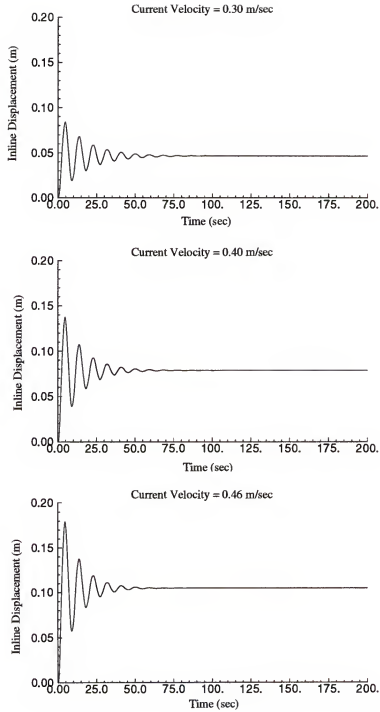


Figure 6.8: Inline Displacement Due to Inline Current at the Middle Node of Riser According to Change of Time. The System Includes Internal Tension and Internal Flow of 6.0 m/sec.

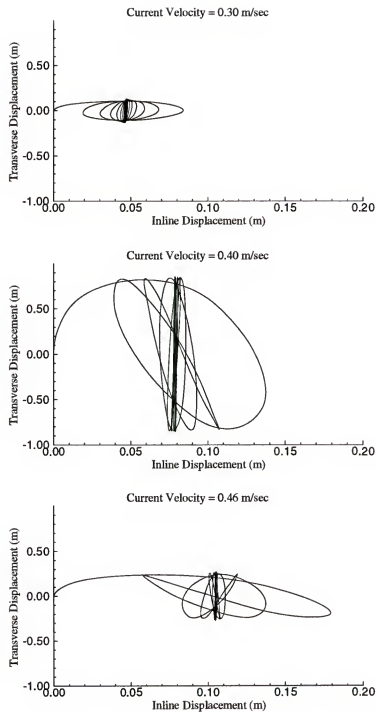


Figure 6.9: Time-Dependent Trajectories of Displacement Vector Tip at the Middle Node of Riser. The System Includes Internal Tension and Internal Flow of 6.0 m/sec.

in the displacement trend between transverse direction and inline direction are also found in Fig. 6.9. It can also be recognized that the trajectory shape changes from almost closed oval to irregular zigzag. This phenomenon results from the shortening of the response period due to the increase of vortex shedding frequency as shown in Fig. 6.7.

Finally, the deflected shapes of the riser system including and excluding internal flow and tension are plotted in Figure 6.10. Four different combinations of plots are given in transverse and current direction. The deflected shapes are drawn at time = 100 sec. the coriolis force effect is seen to be negligible on the inline displacement. This is due to the fact that the response of the riser system in current direction approaches to static steady state after a few oscillations as shown in Fig. 6.8. In other words, 100 sec is enough time for the system to reach a static state such that time-dependent coriolis force effect disappears. Thus, for the case with internal flow inside the riser system but with no internal tension, the curvature of the deflected shape in the current direction simply increases because the centrifugal force due to internal flow leads to a decrease on the stiffness. On the other hand, the riser displacement in inline direction is drastically reduced if internal tension is applied to the system. It is natural because the stiffness term in Eq. (3.63) has to be increased due to internal tension. However, the fact stated above is not always true for the response in the transverse direction because the riser motion is always oscillating regardless of the passage of time. The coriolis force and the centrifugal force both appear to have an effect on the vortex-induced vibration of the riser system all the time whenever the internal fluid flows inside the riser. More explanations are given in chapter 7. The deflected shapes for the case of no internal tension are very different from those with internal tension because the existence of the internal tension changes the system frequency. Therefore, the riser system seems to behave as expected from physical point of view.

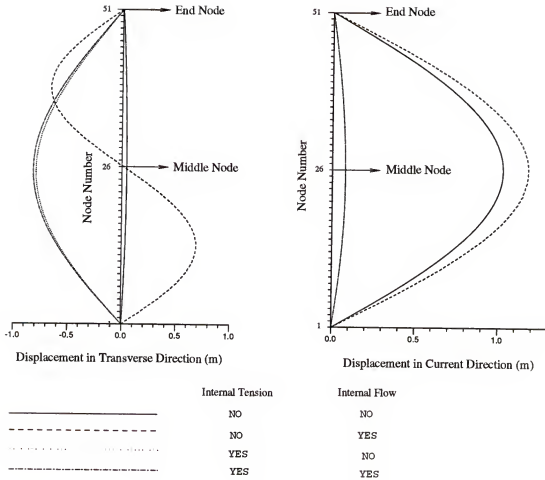


Figure 6.10: Deflected Shape of Riser System in Transverse and Current Direction According to the Change of System. Four Different Combinations Are Considered As Described in Figure. (Current Velocity = 0.40 m/sec, Internal Flow Velocity = 6.0 m/sec)

6.4 Vibration Due to Current or Wave Loading

The matrix equilibrium equation (3.63), which represents inline vibration due to current, can be utilized to obtain the responses of riser under simple current or wave loading because the inline vibration is not coupled with the transverse vibration which is self-excited. In this case, the CODEV is applied by condensing the motion in the transverse direction and some of routines are slightly modified in case of wave loading. The unidirectional wave induced particle velocity and acceleration are obtained in subroutine RWAV based on linear wave theory. Furthermore ELEMV, which calls RWAVE, should also be modified to include the forcing term due to wave and variable CONS(J,I) in routine RCON should be changed into the different type of constants as written below.

CONS(J,I) = input constants and velocities; I = element number

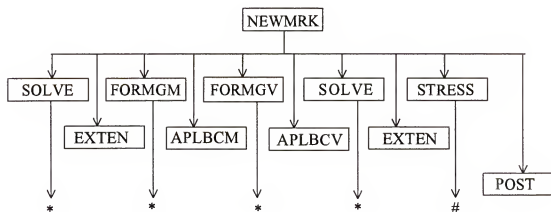
J = 1; the velocity of internal 2; current velocity
 3; current direction to x-axis
 4; Cd 5; Ca
 6; wave height 7; wave number
 8; water depth
 9; wave direction to x-axis
 10; gravity

As described above, CODEV is modified and given as CODEL.

The application of CODEL for the dynamic analysis of the riser system subject to simple current or wave loading does not always give good result because it is programmed on the basis of linear model developed in section 3.5. Since the geometric nonlinearities due to large structural deflections become significant particularly for long risers over 1000 m, the predictor-corrector scheme described in section 6.2 should be implemented to solve the incremental form of the matrix equilibrium equation. The algorithm helps to avoid the possibility of divergence due to iterations and to improve convergence rate. CODE is written for this purpose and NEWMRK, one of the routines in CODE, accomplishes the solution algorithm employing the predictor-corrector scheme

as depicted in Fig. 6.3. Figure 6.11 depicts the flow chart of components of routine NEWMRK. Routine FORMGM and APLBCM is included in NEWMRK because the system mass, damping, and stiffness matrices are dependent on the riser deformations which changes with time and for every iteration step. Routine EXTEN calculates the vertical degrees of freedom using extensibility conditions (See Eqs. (6.12) and (6.13)). In subroutine NEWMRK, routine FORMGM through STRESS are included within the iteration loop and are called upon for every time step. The step calculating the internal tension is performed within the routine STRESS.

Verification on CODEL is not carried out as it is only slightly modified from CODEV keeping the same basic frame. Instead, the CODE version is tested by examining whether the results are reasonable. For the purpose of investigating the validity of CODE, the comparison of the results obtained from CODEL with one from CODE is given. It is well known from previous studies that the nonlinear effects due to large structural deflections and nonlinear drag damping become significant as riser length increases



* : See Figure 4.2

: See Figure 6.4

Figure 6.11: Flow Chart of Components of Subroutine NEWMRK in CODE

or internal tension decreases. On the contrary, the nonlinear effects disappear for the riser with high tension and short length. In this section, CODE and CODEL are both run using the same riser with parameters listed in Figure 5.2, and then compared with each other to see the difference between the linear and nonlinear models due to the change of riser length. The flow field is generated by a linear wave and a uniform current at right angle to each other.

Figure 6.12 and 6.13 plot time simulations of displacement at the middle of the riser in current and wave direction, respectively. Two different length of risers, one long and one short, are used with same top and bottom tension. From those figures, the reduction of nonlinear effect on the short one can be seen. Also, from those time simulation plots, the nonlinear effect in the long riser causes change of the system frequency. The displacement envelopes at a specific time are presented in Figure 6.14 and 6.15 for current and wave direction, respectively. The envelopes in current direction are plotted at time = 200 sec after the riser reaches the static state, while the envelopes in wave direction are plotted at an arbitrary time when the displacement at the middle point of riser is near the positive peak. The difference between linear and nonlinear models grows as riser length increases. The nonlinear effect appears to stiffen the riser for current loading but soften it for wave loading. These trends are also found from previous study (Bernitsas and Kokarakis, 1987). Figure 6.16 shows the maximum displacement in the wave direction according to wave frequency. Like a vortex-induced vibration, the response in the wave direction also shows resonant peaks near the system frequencies but the magnitudes are drastically reduced for higher frequency wave. This phenomenon seems to result from the fact that the wave velocity distribution in water depth decays faster for shorter frequency waves. Subsequently, time histories of the displacement at the middle of a 152 m riser in wave direction are presented in Fig. 6.17 for three wave frequencies near first peak. The response reach a steady oscillating state within a few cycle and shows irregular

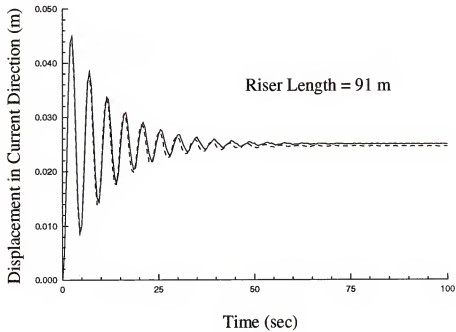
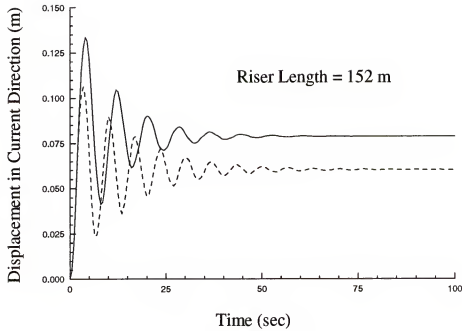


Figure 6.12: Time Simulations of the Displacement at the Middle of Riser in Current Direction. Risers with Different Length Are Subject to a Uniform Current ($U = 0.4$ m/sec) and a Linear Wave ($H_w = 6.1$ m and $T_w = 7.8$ sec) with a Right Angle at the Same Time. (Dashed Line = Nonlinear Model, Solid Line = Linear Model)

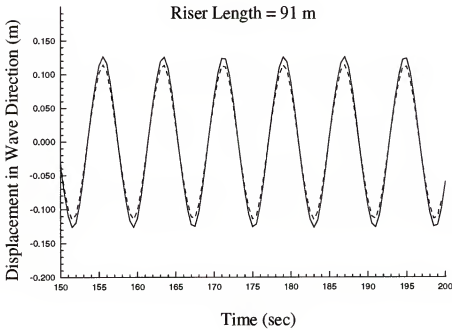
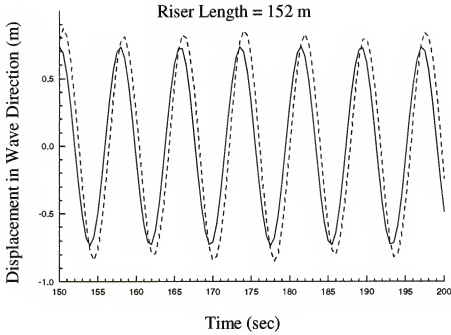


Figure 6.13: Time Simulations of the Displacement at the Middle of Riser in Wave Direction. Risers with Different Length Are Subject to a Uniform Current ($U = 0.4$ m/sec) and a Linear Wave ($H_w = 6.1$ m and $T_w = 7.8$ sec) with a Right Angle at the Riser Tip. (Dashed Line is Nonlinear Model and Solid line is Linear Model)

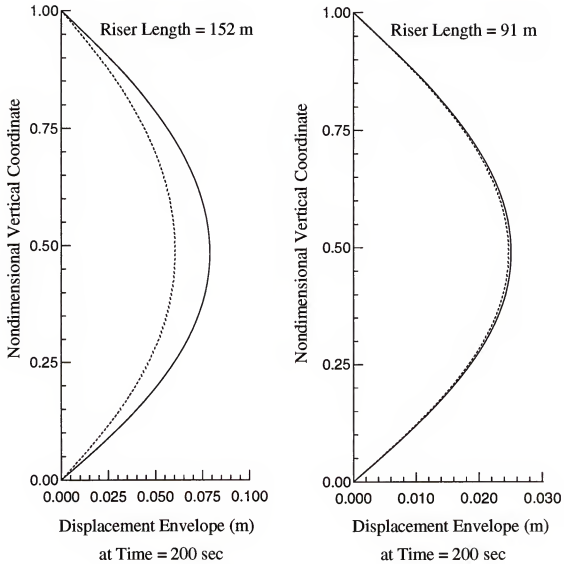


Figure 6.14: Displacement Envelope in Current Direction under Current ($U = 0.4$ m/sec) and Wave ($H_w = 6.1$ m and $T_w = 7.8$ sec) with a Right Angle. (Dashed Line = Nonlinear Model, Solid Line = Linear Model)

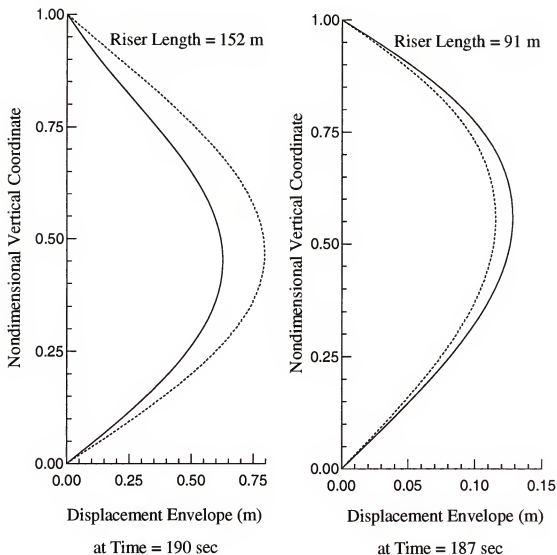


Figure 6.15: Displacement Envelope in Wave Direction under Current ($U = 0.4$ m/sec) and Wave ($H_w = 6.1$ m and $T_w = 7.8$ sec) with a Right Angle. (Dashed Line = Nonlinear Model, Solid Line = Linear Model)

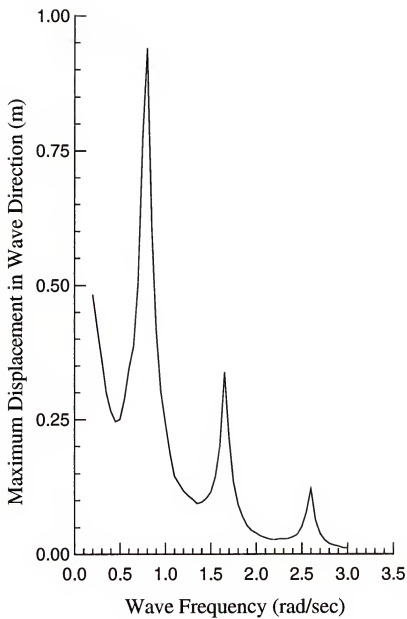


Figure 6.16: Maximum Displacement in Wave Direction According to the Change of Wave Frequency. ($H_w = 6.1$ m)

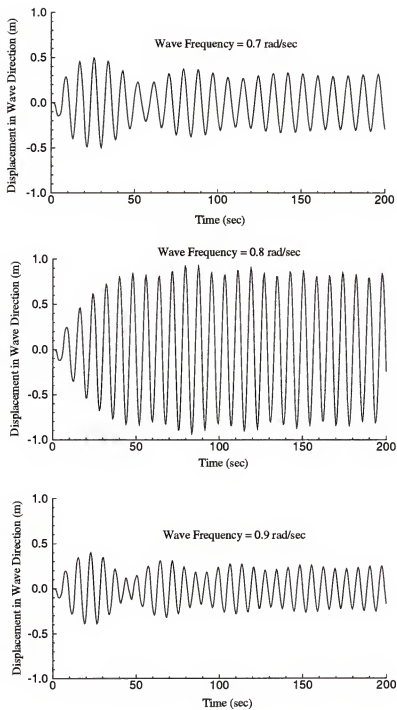


Figure 6.17: Time Simulations of the Displacement at the Middle of a 152 m Riser in Wave Direction. (Hw = 6.1 m)

modulation in amplitude due to the nonlinearities. Time trajectories plotted for the above three cases are very irregular as seen in Fig. 6.18; this irregularity may be due to the combined effect of current and wave applied on the same time. However, if the current approaches the riser at a right angle to wave after the riser response due to wave loading reached a steady oscillating state, the irregular shapes should change to that shown in Figure 6.9.

Even though the structural damping is not still considered in the model, we can easily count for a Rayleigh type damping such as given in Eq. (6.15).

$$[C] = A_0 [M] + A_1 [K] \quad (6.15)$$

where A_0 and A_1 are Rayleigh damping coefficients.

Rayleigh damping can be implemented on CODE simply by adding a few line in the end of routine ELEM. Considering the structural damping, time history and trajectory of displacement at the middle of riser in both direction are presented in Figure 6.19. In this figure, the riser responds to current at a right angle to wave direction after a steady oscillating state in wave direction has been reached and the top tension ratio ($TR = \text{Top Tension} / wL$) is changed from 2.6 to 1.2 for the reduction of stiffness term. Since a high velocity of current ($U = 1.5 \text{ m/sec}$) and a reduced top tension ($TR = 1.2$) are selected, it can be seen clearly from the first figure of Fig. 6.19 that the oscillation of riser in current direction reaches a static state within 2 to 3 cycles with the longer period. In addition, it can be recognized from the third figure of Fig. 6.19 that the time trajectory shape is transformed into more regular shape because of a wave with relatively low period ($T_w = 4 \text{ sec}$) in this application. This trend of time trajectory can be also found in McNamara's study (1988).

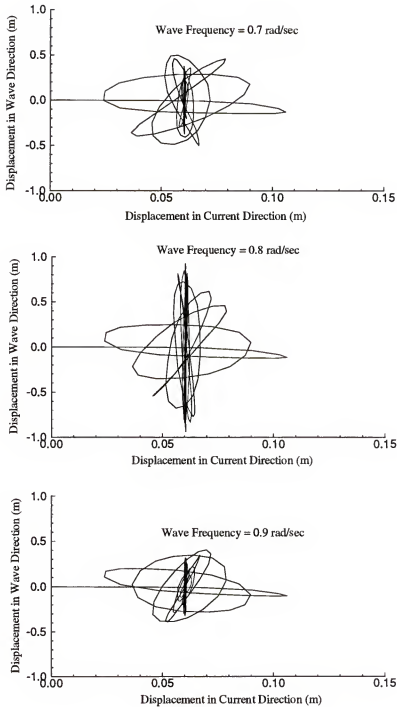


Figure 6.18: Time Trajectories of the Displacement at the Middle of a 152 m Riser under Wave ($H_w = 6.1$ m) and Current ($U = 0.4$ m/sec) with a Right Angle.

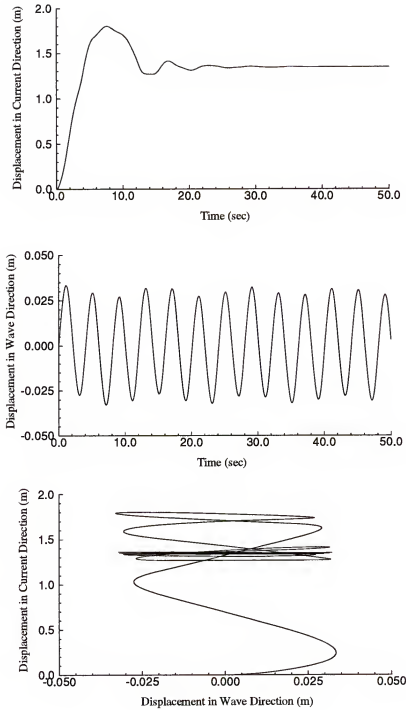


Figure 6.19: Time Simulations in Current and Wave Direction and Time Trajectory at the Middle of a 152 m Riser under Current ($U = 1.5$ m/sec) and a Linear Wave ($H_w = 6.1$ m and $T_w = 4$ sec) with a Right Angle. Rayleigh Damping $[C] = 0.01 [M] + 0.03 [K]$ and Top Tension Ratio $TR = 1.2$

6.5 Modification of System Matrices or Vector for Top Boundary Condition

As mentioned in section 2.5.2, the top boundary condition can be modeled by using translational or rotational springs to provide restoring boundary forces or moments. In addition, structures like a platform or an anchor can be modeled as if mass and damper are attached at the top of riser. These concentrated masses, dampers, and springs are simply added to the corresponding system mass, damping, and stiffness matrix. However, three directional components of top tension should be added to the corresponding system vector. As shown in Eq. 2.53 through 2.55, these direction cosines have deformation-dependent features. Thus, the direction cosines should be included within the iteration routine. The deformation dependency of the direction cosines lost in linear model and therefore, the vertical top tension should be added in the corresponding degree of global stiffness matrix. By the principle introduced above, the modification of system matrices is performed to include the attached properties at the top boundary in subroutine MODGMB.

The numerical application has been made using the riser with a massive structure and mooring system attached at the top. The mass, damping, and spring constants are arbitrarily selected only for the illustrative purpose. The arbitrary selection, however, does not affect the nature of results obtained. The uniform current is applied over the entire riser at a right angle to wave direction after the wave-induced vibration reaches a steady state. The time simulations of tip displacement in wave and current direction are plotted in the first and second figure of Fig. 6.20, respectively and the trace of riser tip on a horizontal plane is plotted in the third. Since the strong mass and damping are attached at the top of riser, it takes long for the riser response in both directions to reach a steady state. In addition, the nonlinear drift in the wave direction can be found even though a linear wave is applied along the riser. Figure 6.21 shows the displacement envelopes in the current direction. It can be recognized from this figure that the riser response reaches a static

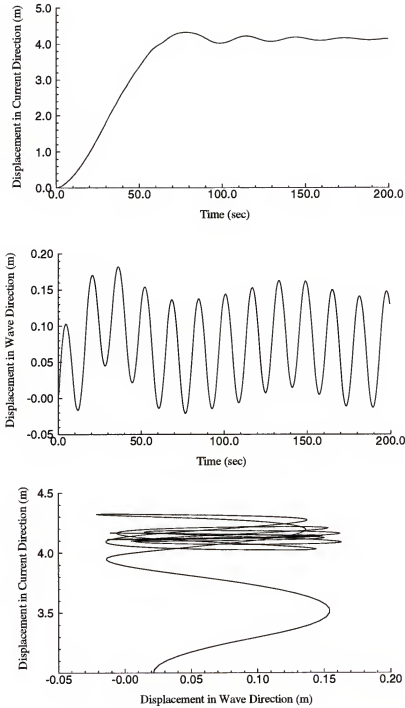


Figure 6.20: Time Simulations in Current and Wave Direction and Time Trajectory at the Top of a 152 m Riser under Current ($U = 1.5$ m/sec) and a Linear Wave ($H_w = 10$ m and $T_w = 16$ sec) with a Right Angle. Rayleigh Damping $[C] = 0.01 [M] + 0.03 [K]$ and Top Tension Ratio $TR = 1.2$

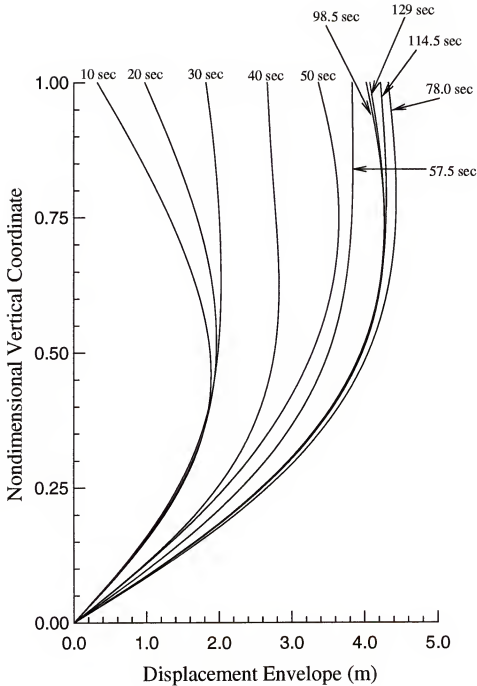


Figure 6.21: The Displacement Envelopes in Current Direction at a Specific Time for a 152 m Riser under Current ($U = 1.5$ m/sec) and a Linear Wave ($H_w = 10$ m and $T_w = 16$ sec) with a Right Angle. Rayleigh Damping $[C] = 0.01 [M] + 0.03 [K]$ and Top Tension Ratio $TR = 1.2$

state. However, it can be seen from Fig. 6.22 that the motion of top does not reach steady state because wave period is relatively short to the period of the riser system. In addition, the nonlinear drift in the wave direction can be also found even though a linear wave pass through the riser.

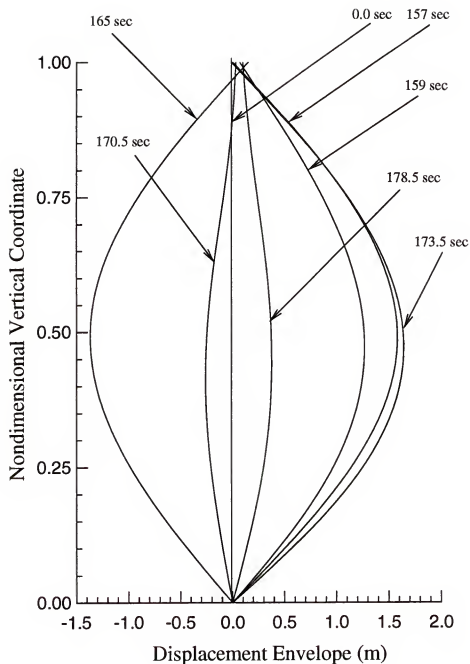


Figure 6.22: The Displacement Envelopes in Wave Direction at a Specific Time for a 152 m Riser under Current ($U = 1.5$ m/sec) and a Linear Wave ($H_w = 10$ m and $T_w = 16$ sec) with a Right Angle. Rayleigh Damping $[C] = 0.01 [M] + 0.03 [K]$ and Top Tension Ratio $TR = 1.2$

CHAPTER 7 THE INVESTIGATION OF THE EFFECT OF INTERNAL FLOW ON RISER DYNAMICS

In the previous chapter, the verification on all the programs developed in this paper has been given. In this chapter the effects of internal flow on riser dynamics such as system frequencies, vortex-induced vibration, and vibrations due to current or wave loading are investigated by applying the numerical model. The investigations are carried out by inspecting how internal flow affects the kinematics and the dynamics of a riser, in particular, the effects of the forcing terms due to the internal flow, namely, the centrifugal and the coriolis forces. In addition, the effect due to the inclusion of the geometric and fluidic nonlinearities are discussed.

7.1 The Effect on System Frequency

For the deep sea application, the riser system is, in general, designed to be nearly neutrally buoyant. Under such condition, the effective weight, the difference between riser weight and local net buoyancy, which results from the combined fluid action of external and internal hydrostatic pressure, is equal to zero, that is, the balance between riser weight and local net buoyancy is locally maintained for the entire length of the riser. The basic parameters governing the property of the riser are the same as used in chapter 5. A change is made on the magnitude of top tension and riser length. The system frequencies were computed for different water depths with six different internal flow conditions and three different top tensions. Figure 7.1 shows the relationship between water depth and system frequency of a neutrally buoyant riser system, i.e., $w = 0$. The ordinate is the ratio of the 1st mode frequency to the natural frequency of the system with null internal flow. As can be seen in the figure, the system frequency is drastically affected by

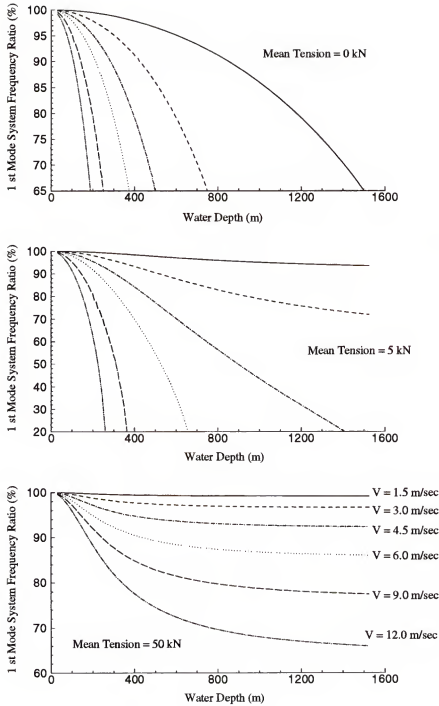


Figure 7.1: Internal Flowing Fluid Effect on 1st Mode System Frequency of a Neutrally Buoyant Riser System. The internal flow velocity changes from 1.5 m/sec to 12.0 m/sec and mean tension (same as top tension) does from null to 50 kN.

the internal flow for the null tension case. Finally, riser buckles when the flow velocity approaches a critical velocity. The increase of top tension, on the other hand, counters the internal flow effect as seen on the second and third figures. With adequate top tension, the system frequency approaches constant as water depth increases. This is because the influence of the stiffness term becomes negligible. Figure 7.2 shows the separate effects of the two flow-induced force components centrifugal and coriolis forces on a riser system. The case is one of the cases in Figure 7.1, with a mean tension of 50 kN and an internal flow velocity of 12 m/sec. From Figure 7.2, it can be seen that the main effect on system frequency is due to centrifugal force. However, even though for cases shown the coriolis force seems to have a minor influence on system frequency, it remains unanswered on its effect on system dynamics such as displacement and stress.

7.2 Comparison between the Effects of Centrifugal and Coriolis Forces

As discussed earlier, internal flow, as it travels along the curved path in the riser, generates centrifugal and coriolis force. These dynamic forces exerted against the riser, in turn, affect the dynamic behavior of a riser. The centrifugal force reduces the stiffness of a riser whereas the coriolis force causes the dynamic coupling with other forces. Their effects on riser behavior are separately examined here. Figure 7.3 and 7.4 present the displacement shapes of a riser at four different arbitrarily selected times to illustrate how the deflected shape changes with the inclusion of the centrifugal force only. In both cases the riser is hinged at the top. Figure 7.3 shows the case under a 5 sec excitation wave and Figure 7.4 shows the case under a 10 sec excitation wave. In comparison with the no internal flow situation, the magnitude of the displacement envelopes are changed but the shapes remain undistorted. In other words, the ratio of magnification remains constant along the riser. Also, it should be noted here, since centrifugal force reduces the stiffness of the riser, the amplitude of the riser displacement also increases with increasing centrifugal force. However, a phase difference is introduced into the system. Therefore,

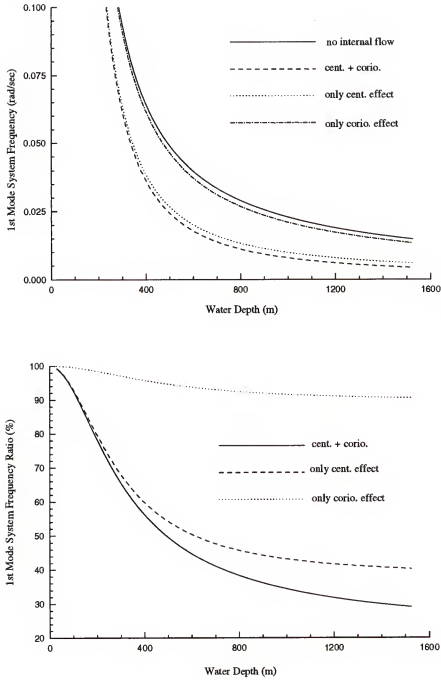


Figure 7.2: Comparison Between the Effects of Each Forcing Terms (Centrifugal and Coriolis Forces) Due to Internal Flow on 1 st System Frequency. Mean Tension = 50 kN, Internal Flow Velocity = 12 m/sec

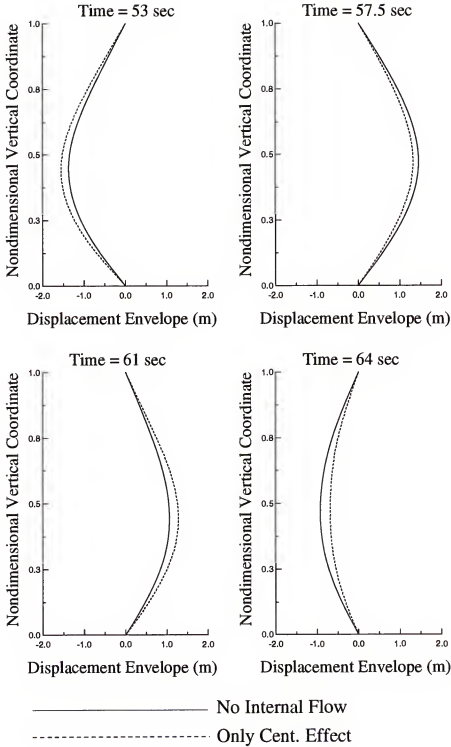


Figure 7.3: The Effect of Centrifugal Force on Displacement Envelope of Riser at an Arbitrary Time. ($T_R = 1.2$, $w = 3.86$ kN/m, $H_w = 6.1$ m, $T_w = 15$ sec)

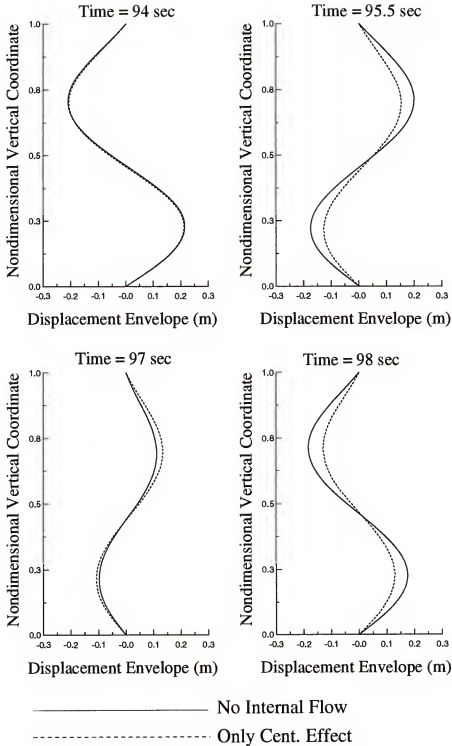


Figure 7.4: The Effect of Centrifugal Force on Displacement Envelope of Riser at an Arbitrary Time. ($T_R = 1.2$, $w = 3.86$ kN/m, $H_w = 6.1$ m, $T_w = 5$ sec)

the amplitude amplification will not be manifested at every instance.

Unlike the effect of centrifugal force, the coriolis force alters the displacement shape as shown in Fig. 7.5 and 7.6. As can be seen, the magnitudes of displacement change at upper and lower part of riser are different. This can be understood from the fact that the coriolis force term is composed of a mixed derivative with respect to time and space or the coriolis matrix in matrix equilibrium equation is skew symmetric. The coriolis force alternately couples with inertia and stiffness according to the passage of time. In addition, it can be recognized from the comparison of figure 7.5 with 7.6 that its effect is pronounced for shorter period wave excitations. It results from the fact that the coriolis force is a function of rotational velocity which is faster under shorter period waves.

In summary, the centrifugal force, which depends only on the curvature of riser deflection, does not alter the displacement shape, while the coriolis force, which has a mixed derivative of time and space, distorts the displacement shape. Further, it can be inferred from the figures in this section that the system frequencies are affected by both forces.

7.3 The Effect on Vortex-Induced Vibration

As described in the previous chapter, Vortex-induced vibration due to inline current has been formulated by modifying Iwan-Blevin's model to include the effect of internal flow and a computer program CODEV has been developed to solve it. The transverse vibration due to vortex shedding with the inclusion of internal flow is investigated here, while the inline vibration in current direction will be discussed in section 7.5.

The time simulation of the displacement at the middle node of a riser is presented in Figure 7.7. The presence of internal flow in this case, which has a very high top tension, causes a slight increase in amplitude and a slight shift in phase. Figure 7.8 presents the trajectory of displacement at this middle node according to the lapse of time. The

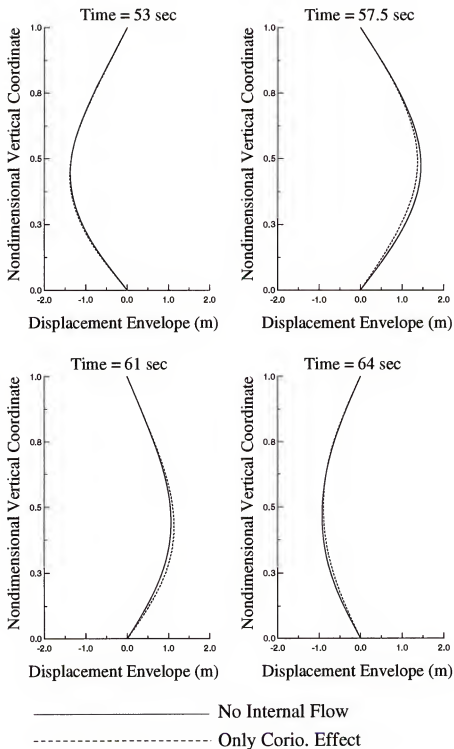


Figure 7.5: The Effect of Coriolis Force on Displacement Envelope of Riser at an Arbitrary Time. ($TR = 1.2$, $w = 3.86$ kN/m, $H_w = 6.1$ m, $T_w = 15$ sec)

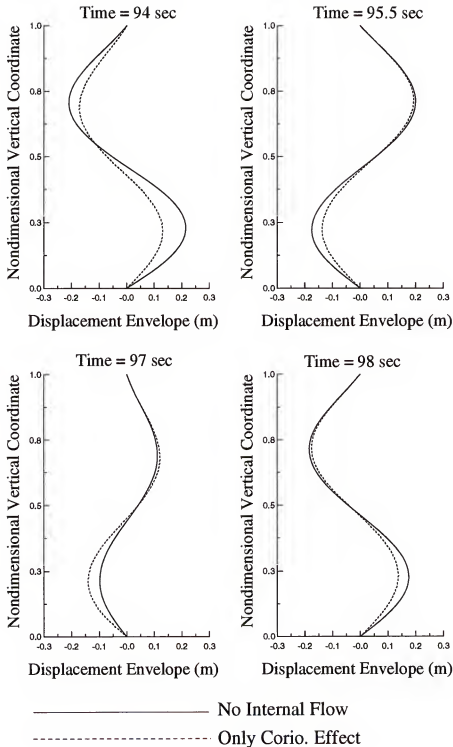


Figure 7.6: The Effect of Coriolis Force on Displacement Envelope of Riser at an Arbitrary Time. (TR = 1.2, w = 3.86 kN/m, Hw = 6.1 m, Tw = 5 sec)

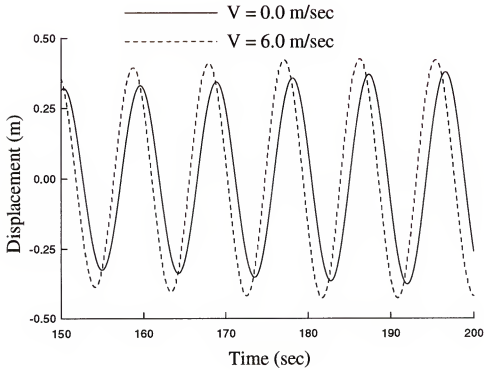
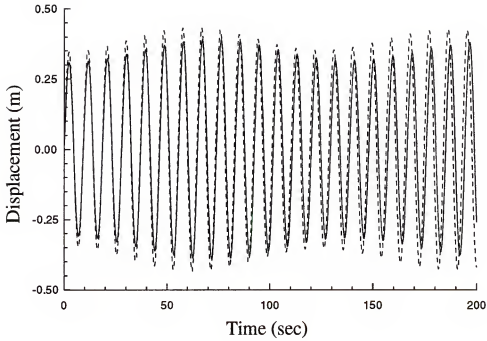


Figure 7.7: The Effect of Internal Flow on Displacement at the Middle of 152 m Riser. (Current Velocity $U = 0.32$ m/sec, Top Tension = 1550 kN, The Other Properties are the Same as Example of Fig. 7.6)

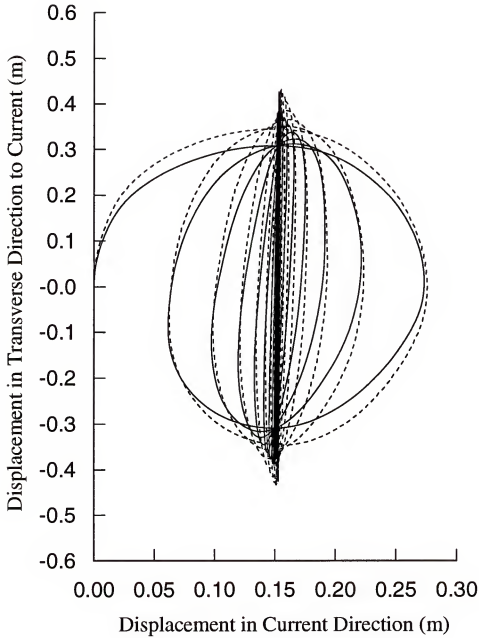


Figure 7.8: The Effect of Internal Flow on the Trajectory at the Middle of 152 m Riser According to the Lapse of Time. (Current Velocity $U = 0.32$ m/sec, Top Tension = 1550 kN, The Other Properties are the Same as Example of Fig. 7.6)

trajectory shape is modified more so in the transverse direction than the inline direction. This is partly due to the high tension applied at top of riser. As top tension eases, the effect of internal flow on the riser responses, particularly on inline displacement, should become more pronounced.

The optimal riser tension can be determined by minimizing the combined stress of the axial stress resulting from internal tension and the bending stress from lateral deflection. Thus, although high top tension tends to reduce the riser displacement and the associated bending stress, there is a limit to its application because the increase in tensile stress may cut the benefit on the reduction in bending stress. Since direct stress increases with water depth, top tension has to be eased for deep water application. It is known from previous investigations that the effect of internal flow on riser dynamics is not significant under high top tension. However, the question is whether the internal flow effects should be considered for deep water case for the reason give above.

For comparison purpose, the same riser in a 152 m of water depth as used in previous examples but with very low top tension is used here. Computations are then performed for three types of tension (high, middle, and low), each with three internal velocities (null, high, and very high). Figure 7.9 shows the effect of internal flow on maximum displacement in the transverse direction, i.e., the direction that the forces due to vortex shedding act. Since the vortex model proposed by Iwan and Blevin has proven to work well in certain range of Reynolds number, current velocities selected in the present computations are also determined from equation (2.72) within this Reynolds number range. As shown in the figure, the presence of internal flow causes only a slight shift of the resonant bands as well as a slight increase in peak values. However, for non-peak positions, the effect of internal flow on displacement amplitude is much larger. Following the curve of any set one sees that internal flow causes both amplitude amplification and reduction depending upon the flow velocity. This is clearly significant in riser design

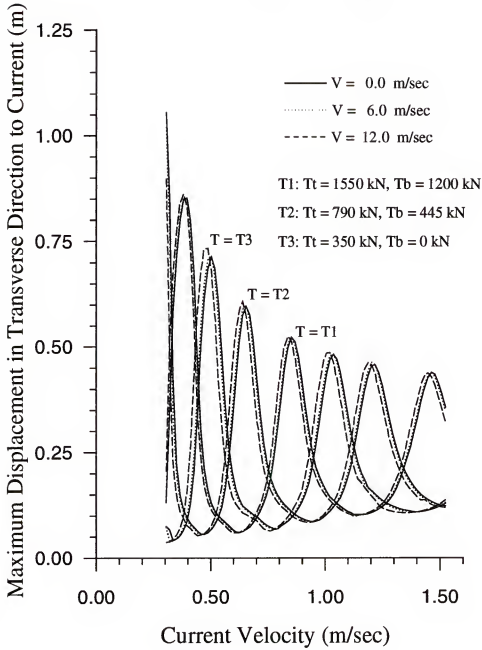


Figure 7.9: The Effect of Internal Flow on Maximum Displacement in Vortex Direction for 152 m Riser. Riser Properties except Top Tension are the Same as Example Figure 7.6.

in determining the optimum internal flow velocity. In other words, displacement amplitudes are either augmented or reduced depending on where the current velocity magnitudes as shown in this figure. This situation is also shown in Figures. 7.10 and 7.11. Also, as internal current velocity increases, the modal shape will also shift from lower to higher modes. For instance, for the case presented in Figure 7.10, the riser deflection shape changes from 1st to 2nd mode when $U = 0.6$ m/sec and, again, changes to 3rd mode when $U = 0.9$ m/sec.

In summary, the effect of internal flow on vortex-induced vibration is not negligible, in particular, when the top tension is low as the cases found in deep water.

7.4 Consideration of Geometric Nonlinearity

The nonlinearities in riser dynamic analysis are mainly of two origins that from the flow-induced drag force and from geometric nonlinearities due to large structural deflections. In particular, the latter becomes significant for long riser. To examine the effects of geometric nonlinearity maximum displacement are of riser with internal flow can be computed by the program for different riser lengths or water depths. But, there is no readily available information on selecting riser parameters for every water depth and on determining top tension value to avoid riser failure due to excessive stress. A conservative approach is taken here to let the top tension maintained a constant regardless of water depth and then, let the effective weight be adjusted to avoid negative tension over the riser length for different water depths. In practice, this effective weight can be adjusted by altering the buoyancy along the riser.

With this hypothetical riser, the maximum displacements of the riser are computed for water depth up to 1500 m using both linear and nonlinear models. The ratios of these maximum displacements to that of the case without internally flow are plotted in Figure 7.12. As shown in the figure, this maximum displacement ratio basically increases with depth to a certain peak point beyond which it decreases with depth and gradually

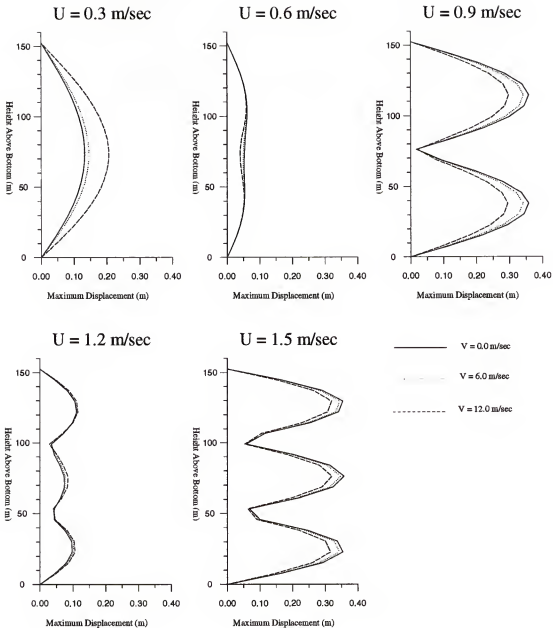


Figure 7.10: The Effect of Internal Flow on Maximum Displacement Envelope in Vortex Direction for 152 m Riser at Some Current Velocity. Top Tension = 1550 kN, Bottom Tension = 1200 kN. Riser Properties except Tension are the Same as Example of Figure 7.6. Top Condition is Semi-Restrained.

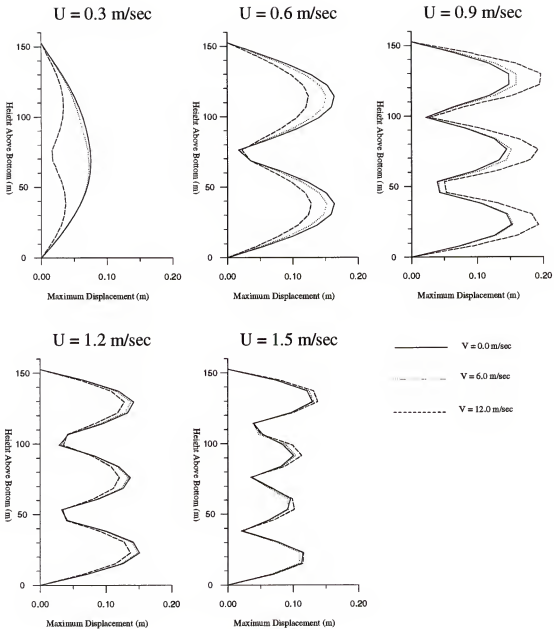


Figure 7.11: The Effect of Internal Flow on Maximum Displacement Envelope in Vortex Direction for 152 m Riser at Some Current Velocity. Top Tension = 350 kN, Bottom Tension = 0 kN. Riser Properties except Tension are the Same as Example of Figure 7.6. Top Condition is Semi-Restrained.

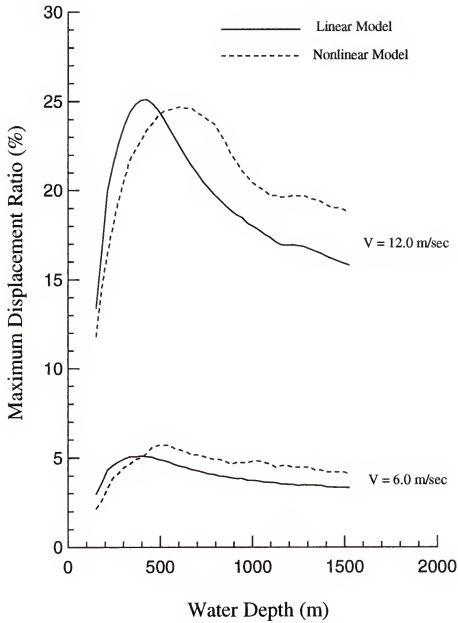


Figure 7.12: Comparison Between Linear and Nonlinear Model about the Effect of Internal Flow According to the Change of Water Depth and Internal Flow Velocity. Top Tension = 650 kN, $TR = 1.1$, Effective Weight = Top Tension / ($TR \times$ Riser Length)

approaches a constant as the effect of the riser stiffness also diminishes with depth. The effect due to nonlinear geometry can also be examined from this Figure. As can be seen, in shallow water the effect of geometric nonlinearity is actually to reduce the displacement ratio. However, as the water becomes deeper than certain critical value, the displacement ratio becomes larger with the consideration of the geometrical nonlinearity. The differences from the linear model and the nonlinear model clearly are not negligible and they become larger for higher internal flow velocity. For the case of increased top tension such as shown in Figure 7.13, the displacement ratio is also found to follow the same trend but the differences between the linear and nonlinear models become smaller.

In summary, the effect of geometrical nonlinearity becomes increasingly more important for risers in deeper water, particularly if the top tension is low and the internal flow velocity is high. Under such conditions, linear model underestimates the displacement as compared to nonlinear model. A linear model can be used if top tension is high which generally restricts its application to shallow water.

7.5 Riser Vibrations Due to Wave or Current Loading

In this section the system vibrations due to current or wave loading of a riser with internal flow are examined. The CODE program developed in this investigation is used to perform the analysis. As mentioned in earlier the CODE program includes the nonlinear effects due to flow-induced drag force and large structural deformation, both of which become increasing important in deeper waters.

First, the time simulations of tip displacements are calculated for the condition of a combined uniform current and a wave system at right angle to each other. The results are shown in Figures 7.14 to 7.16. Figure 7.14 shows the current-direction inline displacement which behaves like a damped oscillations that eventually will approach a steady displacement. Since a substantial mass with strong damping and spring characteristics is attached at the top of riser to model a massive anchored platform, the effect of

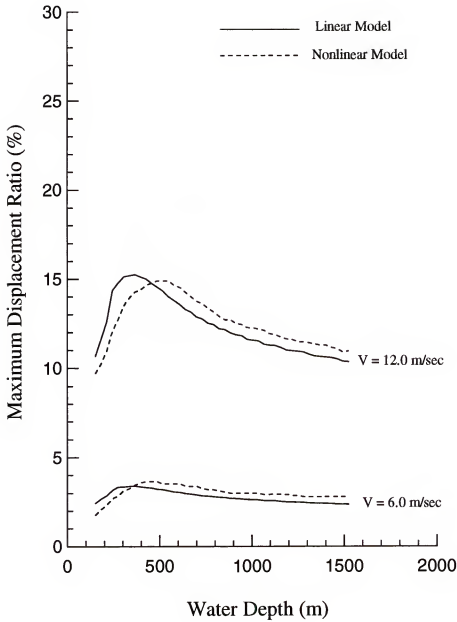


Figure 7.13: Comparison Between Linear and Nonlinear Model about the Effect of Internal Flow According to the Change of Water Depth and Internal Flow Velocity. Top Tension = 765 kN, TR = 1.1, Effective Weight = Top Tension / (TR x Riser Length)

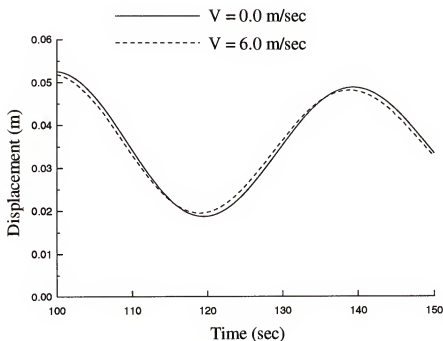
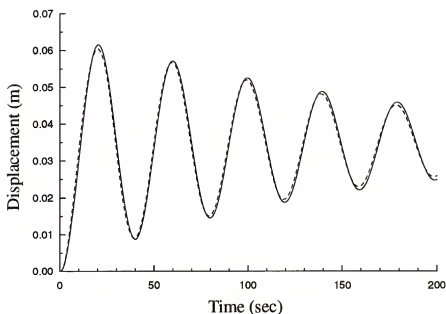


Figure 7.14: The Effect of Internal Flow on the Displacement in Current Direction at the Tip of 152 m Riser. Second Figure Shows the Zoom Picture at Certain Time Interval. (Current Velocity $U = 0.40$ m/sec, Top Tension = 350 kN, Effective Weight $w = 2.3$ kN/m, Semi-Restrained Top Condition)

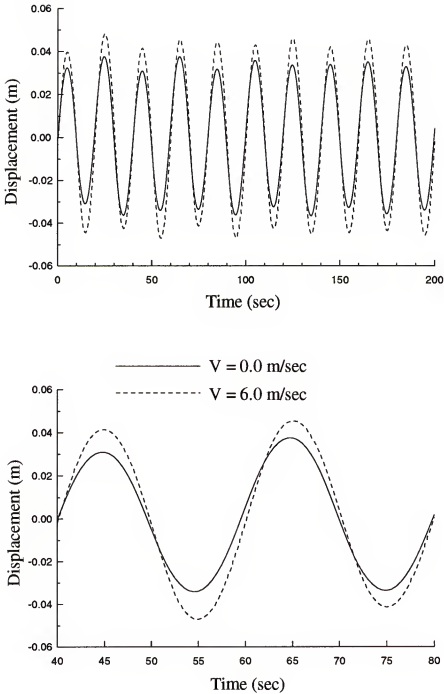


Figure 7.15: The Effect of Internal Flow on the Displacement in Wave Direction at the Tip of 152 m Riser. Second Figure Shows the Zoom Picture at Certain Time Interval. (Wave Period $T_w = 20 \text{ sec}$, Wave Height $H_w = 6.1 \text{ m}$, Top Tension = 350 kN, Effective Weight $w = 2.3 \text{ kN/m}$, Semi-Restrained Top Condition)

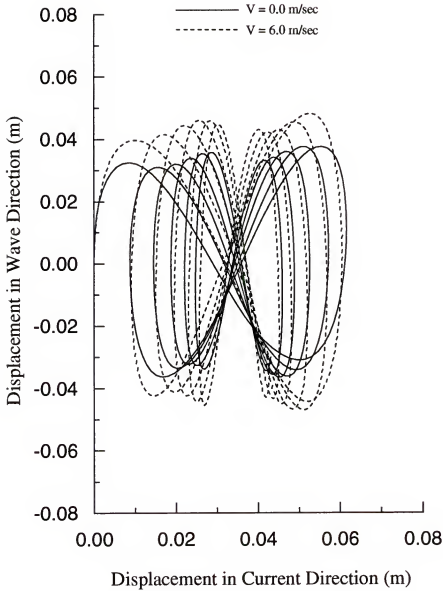


Figure 7.16: The Effect of Internal Flow on the Time-Dependent Trajectory at the Tip of 152 m Riser. (Current Velocity $U = 0.4$ m/sec, Wave Period $T_w = 20$ sec, Wave Height $H_w = 6.1$ m, Top Tension = 350 kN, Effective Weight $w = 2.3$ kN/m, Semi-Restrained Top Condition, Current and Wave Cross with an Right Angle)

internal flow is clearly overshadowed in this case. Figure 7.15 plots the time displacement curve in the wave direction. It is shown the effect of internal flow is more pronounced in this direction owing to the fact that the coriolis force is maintained by the wave oscillation. Figure 7.16 shows the time-dependent trajectory of riser's tip displacement. As expected, the coupled response in both directions results in the narrowing pattern in the current direction but maintains a modulated oscillation in the wave direction. The presence of internal flow causes a slight decrease in current-direction displacement but a more pronounced increases in wave-direction displacement. Figure 7.17 presents the effect of internal flow on the maximum displacement as a function of current strength for different riser tensions and internal velocities. In this case, the centrifugal force dominates the coriolis effect and the effect of internal flow becomes larger with increasing current velocity as a result of the increased riser deformation. This effect is further magnified for risers with low tension as seen in this figure. In other words, in order to reduce the centrifugal force due to internal flow one must increase either the top tension or the buoyancy. These effects also can be found in Figures 7.18 and 7.19 which present, respectively, the cases of internal flow on maximum envelope of riser under high and low top tensions. Under high top tension, the centrifugal force effect is relatively insignificant, while under low tension its effect becomes more important as current velocity increases. Figure 7.20 plots the maximum displacement in the wave direction as a function of wave frequency for different top tensions and internal flow velocities. Like the case of vortex-induced vibration, displacement due to wave loading also has resonance peaks of which the positions are lightly shifted due to the presence of internal flow. The first resonance peak apparently dominates the rest for all the cases. Here, the fundamental natural frequency of the low tension case (about 0.4 rad/sec in Fig. 7.22) is approximately half that of the high tension case (about 0.8 rad/sec in Fig. 7.21). In these first mode frequency bands, coriolis force appears to be more important than the centrifugal force in internal

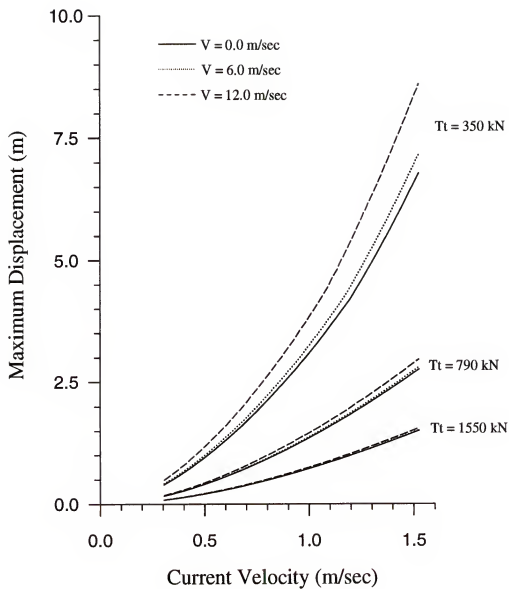


Figure 7.17: The Effect of Internal Flow on Maximum Displacement According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

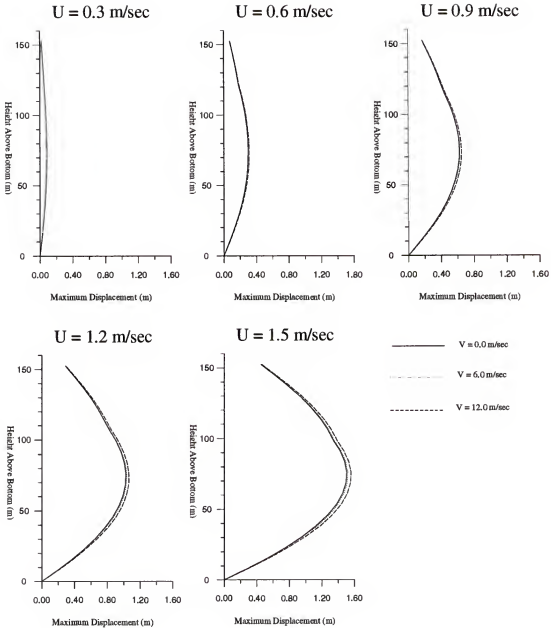


Figure 7.18: The Effect of Internal Flow on Maximum Envelopes of a 152 m Riser According to the Change of Current Velocity under High Top Tension of 1550 kN. System is Same as Example of Fig. 7.16 except Parameters as Shown in This Figure.

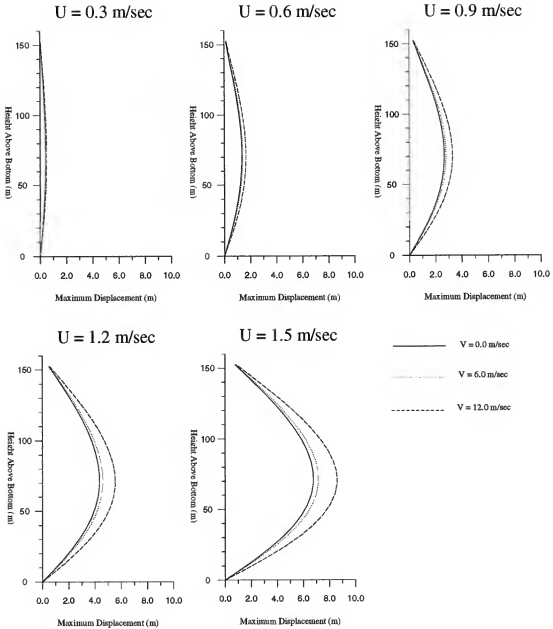


Figure 7.19: The Effect of Internal Flow on Maximum Envelopes of a 152 m Riser According to the Change of Current Velocity under Low Top Tension of 350 kN. System is Same as Example of Fig. 7.16 except Parameters as Shown in This Figure.

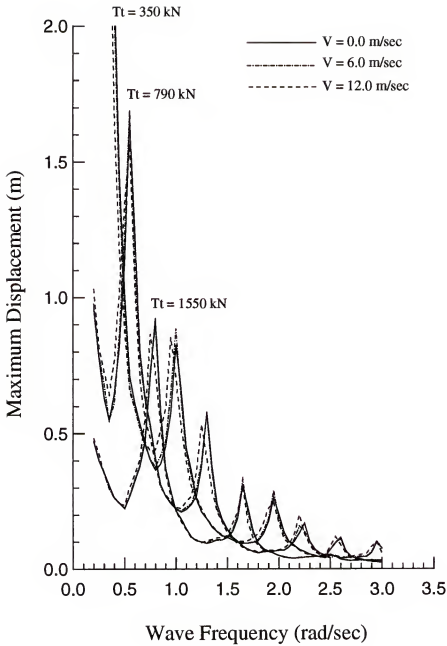


Figure 7.20: The Effect of Internal Flow on Maximum Displacement According to the Increase of Wave Frequency. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

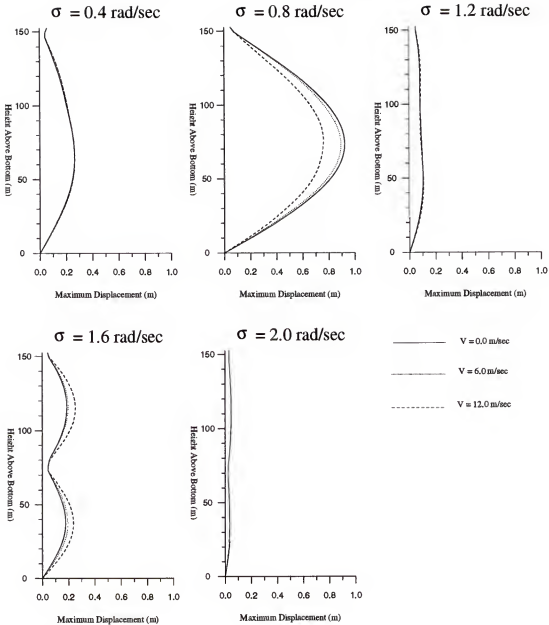


Figure 7.21: The Effect of Internal Flow on Maximum Envelopes of a 152 m Riser According to the Change of Wave Frequency under High Top Tension of 1550 kN. System is Same as Example of Fig. 7.16 except Parameters as Shown in This Figure.

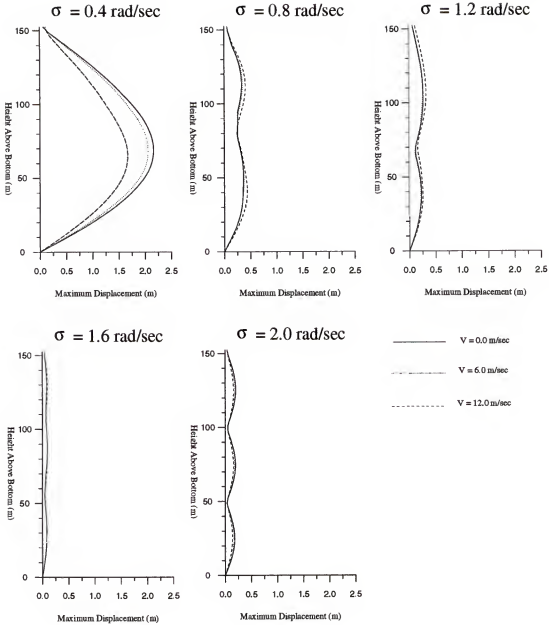


Figure 7.22: The Effect of Internal Flow on Maximum Envelopes of a 152 m Riser According to the Change of Wave Frequency under Low Top Tension of 350 kN. System is Same as Example of Fig. 7.16 except Parameters as Shown in This Figure.

flow effects. This is observed from the reduced peak values due to the positive effect of coriolis force. Centrifugal force should have the adverse effect of increasing the displacement. The same, however, can not be said for higher modes. Figures 7.21 and 7.22 also shows that the internal flow effect, be it coriolis or centrifugal, plays a more important role under low top tension as evidenced from comparing the changes of modal envelopes due to changes of internal velocities in these two cases.

In summary, the effect of internal flow on tip displacement is not significant if the riser is restrained at the top by a large mass with strong damping and stiff spring. The effect of internal flow on the displacement of a riser in a current field is dominated by centrifugal force because the riser response is slowly time varying but could have large curvature. On the other hand, for riser under pure wave loading the internal flow effect is dominated by coriolis force owing to the oscillatory nature of riser response. Also, centrifugal force always increases the displacement which is undesirable, but coriolis force could induce reduced displacement at certain resonant peaks.

7.6 Effects on Internal Stresses and Displacement Fields

In this section, the effects of internal flow on riser kinematics and dynamics such as displacement, rotation, shear force, and bending moment, are investigated. The responses due to current loading and vortex shedding are addressed specifically. The responses due to wave loading are of same nature as due to vortex shedding.

As shown in Fig. 7.23, the effects of internal flow on maximum displacement and rotation become more important when current velocity and/or internal flow velocity increase, and when top tension decreases. In particular, it can be seen from Fig. 7.24 that the maximum effect on displacement occurs at locations where the slopes are zero and the maximum effect on riser rotation occurs at the bottom. The effects on bending moment are the same as on displacement but the effects on shear force are not as evidenced in Figures 7.25 and 7.26. For the system under harmonic loading such as vortex shedding

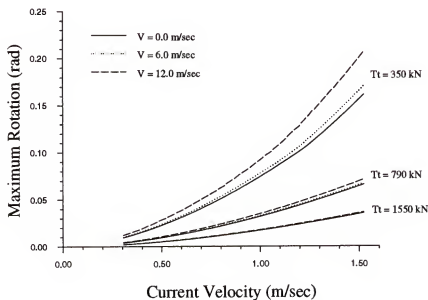
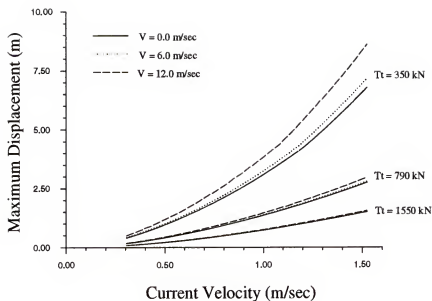


Figure 7.23: The Effect of Internal Flow on Maximum Displacement Fields According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

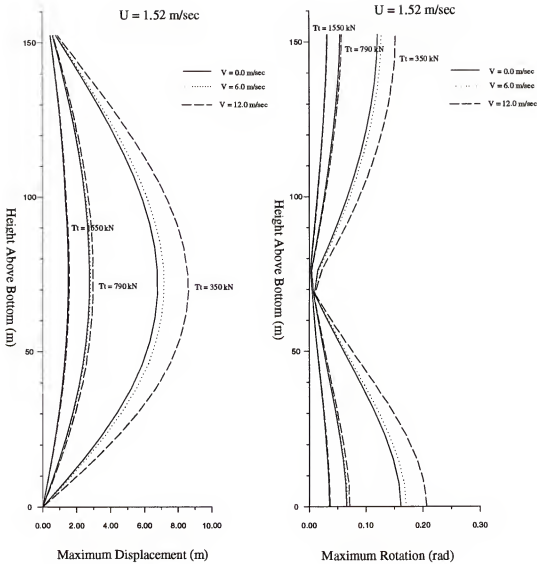


Figure 7.24: The Effect of Internal Flow on Maximum Displacement and Rotation Envelope Due to Current Loading for Three Types of Top Tension. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

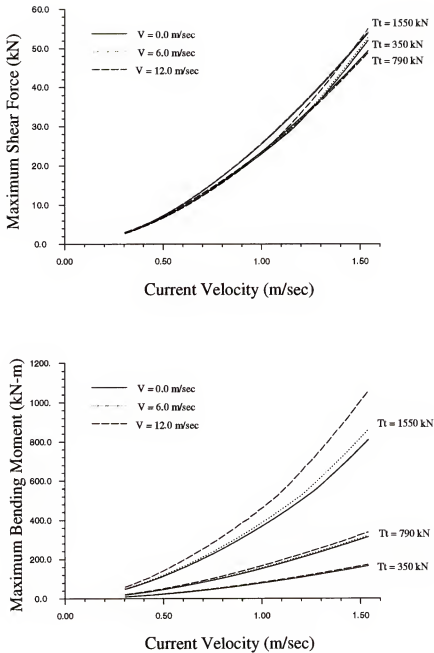


Figure 7.25: The Effect of Internal Flow on Maximum Internal Stress Fields Due to Current According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

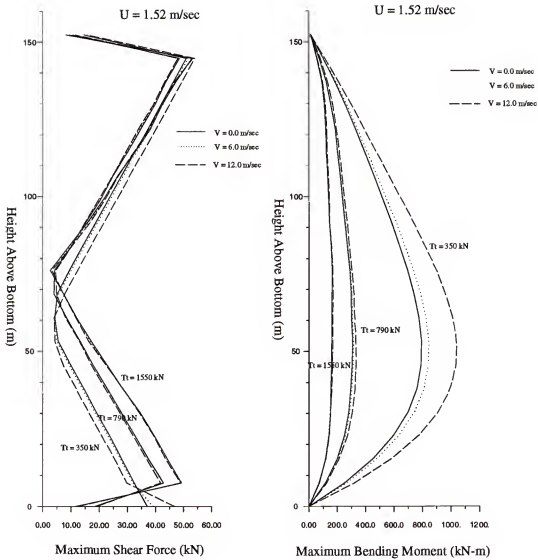


Figure 7.26: The Effect of Internal Flow on Maximum Shear and Bending Moment Envelope Due to Current Loading for Three Types of Top Tension. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

or wave, the resonance bands of both displacement and rotation are affected slightly and the changes become more visible under low tension (See Fig. 7.27). Also, it can be seen from Fig. 7.28 the maximum effect on displacement is at locations where the slope is zero, while the maximum effect on rotation occurs not only at bottom but also at inflection points. Shear force or bending moment also experience changes of resonant bands in the same fashion as the rotation or displacement does (See Fig. 7.29). Furthermore, maximum effect on shear force occurs at inflection points, as shown in Figure. 7.30.

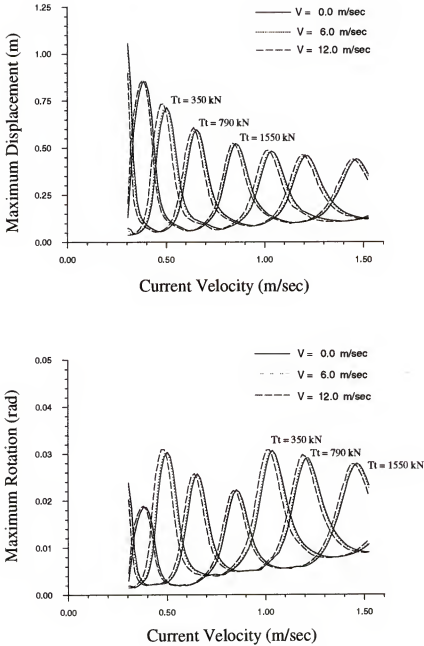


Figure 7.27: The Effect of Internal Flow on Maximum Displacement Fields Due to Vortex Shedding According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

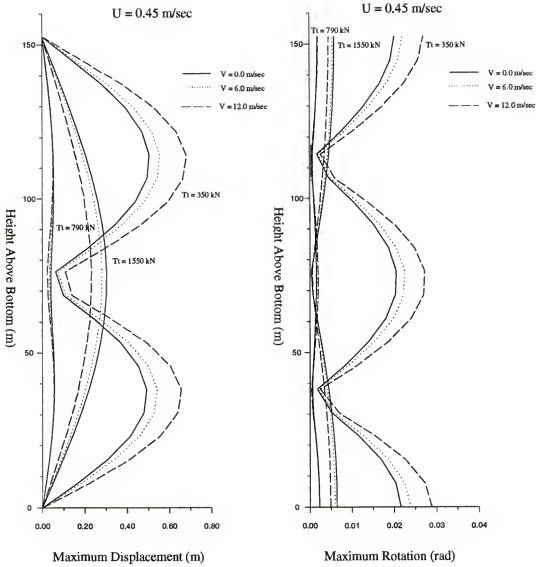


Figure 7.28: The Effect of Internal Flow on Maximum Displacement and Rotation Envelopes Due to Vortex Shedding for Three Types of Top Tensions. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

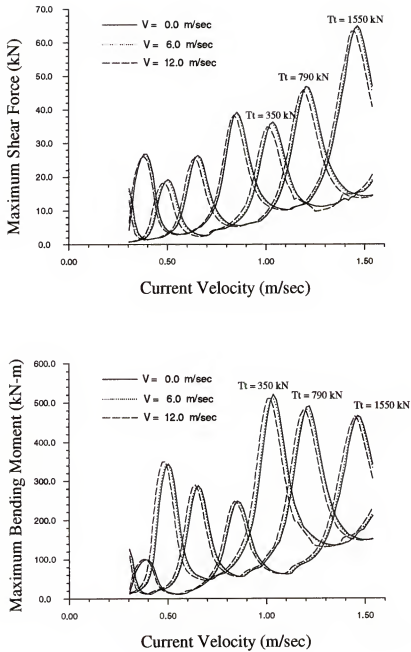


Figure 7.29: The Effect of Internal Flow on Maximum Stress Fields Due to Vortex Shedding According to the Increase of Current Velocity. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

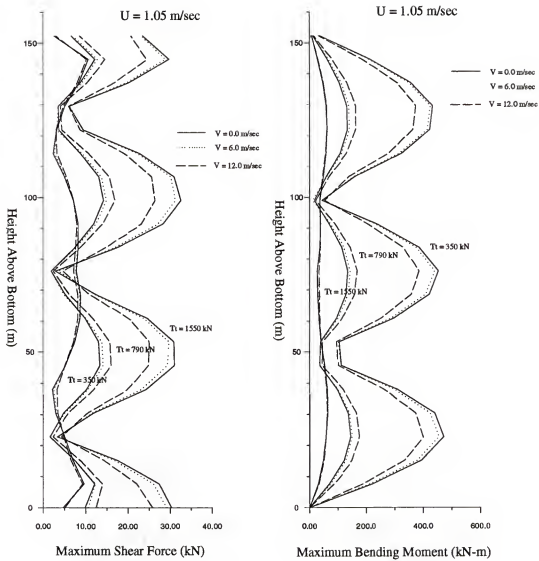


Figure 7.30: The Effect of Internal Flow on Maximum Stress Envelopes Due to Vortex Shedding for Three Types of Top Tensions. System is Same as Example of Fig. 7.16 except Parameters Shown in this Figure.

CHAPTER 8 CONCLUSIONS AND FURTHER STUDY

8.1 Conclusions

A mathematical model for the analysis of riser systems with the inclusion of internal flow is developed together with numerical solutions and computer programs. Four different programs are developed in this study; FRISER program computing the system frequencies by using semi-analytical method, CODE solving a nonlinear model by implementing predictor-corrector scheme, CODEV computing vortex-induced vibrations, and CODEL obtaining the solutions of a linear model. From the results of sample computations, the effects of internal flow on riser dynamics are examined. The following conclusions are drawn:

1) There are two dynamic forces due to the motion of internal fluid, that is, centrifugal and coriolis force. The centrifugal force, which depends only on the curvature of riser deflection, does not alter displacement shape whereas the coriolis force, which is a term with mixed derivative of time and space, distorts the displacement shape.

2) The system frequency is significantly affected by the internal flow for the null tension case. The riser will buckle finally when flow velocity reaches a critical velocity. The increase of top tension, on the other hand, partially counters the internal flow effects. Further, the main effect on system frequency is due to centrifugal force.

3) The effect of internal flow on vortex-induced vibration is not negligible, in particular, when the top tension is low as for the cases in deep water.

4) The effect of geometrical nonlinearity becomes increasingly more important for risers in deeper water, particularly if the top tension is low and the internal flow velocity

is high. Under such conditions, the linear model underestimates the displacement as compared to the nonlinear model. A linear model can be used if top tension is high which generally restricts its application to shallow water.

5) The effect of internal flow on tip displacement is not significant if the riser is restrained at the top by a large mass with strong damping and a stiff spring.

6) The effect of internal flow on the displacement of a riser in a current field is dominated by centrifugal force. On the other hand, for riser under pure wave loading the internal flow effect is dominated by coriolis force. Also, centrifugal force always increases the displacement which is undesirable, but coriolis force could increase or reduce displacement depending on the modes of resonant peaks.

7) The effects of internal flow on maximum displacement and rotation become more important when current velocity and/or internal flow velocity increase, and when top tension decreases. In particular, the maximum effect on displacement occurs at locations where the slopes are zero and the maximum effect on riser rotation occurs at the bottom. The effects on bending moment are the same as on displacement but the effects on shear force are not in certain rule.

8) For the system under harmonic loadings such as vortex shedding or wave, the resonance bands of both displacement and rotation are affected slightly and the changes become more visible under low tension. Also, the maximum effect on displacement is at locations where the slope is zero, while the maximum effect on rotation occurs not only at bottom but also at inflection points. Shear force or bending moment also experience changes of resonant bands in the same fashion as that of the rotation or displacement. As expected, the maximum effect on bending moment occurs at zero slope points and the maximum effect on shear force occurs at inflection points.

In addition to the conclusions made in this study, it should be noted that the effects by coriolis force could be overestimated because downward internal flow is not included

in the construction of a mathematical model even though the downward flow tends to reduce coriolis force.

8.2 Further Study

In this dissertation, Iwan-Blevin's model, which is one of the self-excited oscillator models, has been implemented as a vortex shedding model to examine the effect of internal flow. In that model, the span of riser is divided into segments that are within the resonance band and the remainder, which is outside the resonance band. However, the numerical application of the spanwise extent of resonance band is not feasible due to the distortion of mode shape resulting from the mixed derivative term even though it is expected to be small. Thus, we need to develop the more refined current-vortex model to include the depth-dependent resonance described above. Besides, the nonlinear coupling between inline and transverse vibration and the extension of the applicable range of Reynolds number may be required for the development of the refined model to investigate the effect of internal flow more precisely.

The internal flow causes the distortion of trajectory shape as discussed in the previous chapter. Thus, it may be required to include the internal torsion in the riser analysis. However, the coupling problem with torsion is not easy task. In particular, the torsional-axial coupling problems still remain to solve. Uncoupled internal torsion may be applied in the derivation of model and the effect of internal flow on it may be investigated using the model in the future study.

In addition, random wave excitation and corresponding rig motion may be also included in the future study. An explicit time-dependent representation of wave elevation and velocity may be generated for a time domain.

APPENDIX A DERIVATION OF LINEAR GOVERNING EQUATION USING HAMILTON'S PRINCIPLE

As an element of fluid flows with a velocity V_I along the pipe under the assumption of small deformation, the x -component of velocity is $\dot{x} + V_I x'$. The kinetic energy in a length dz of the pipe is that of the pipe plus that of the fluid, which is

$$dT = \left\{ \frac{1}{2}(m_t - m_f)\dot{x}^2 + \frac{1}{2}m_f[V_I^2 + (\dot{x} + V_I x')^2] \right\} dz$$

The first term on the right side represents the kinetic energy of the pipe and added mass, and the second represents the kinetic energy of the internal fluid. The strain energy due to the lateral deflection in the length dz is

$$dV = \left[\frac{1}{2}EI(x'')^2 - \frac{1}{2}T(x')^2 \right] dz$$

The first term on the right side represents the stiffness strain energy and the second represents the tension strain energy. The negative sign in front of tension strain energy indicates that it reduces the total strain energy due to the lateral deflection. Hamilton's principle states that $\delta \int_{t_1}^{t_2} (T - V) dt = 0$.

Substituting the kinetic energy and the strain energy and performing the variation gives

$$\int_{t_1}^{t_2} \int_z^z \{ (m_t - m_f)\dot{x}\delta\dot{x} + m_f(\dot{x} + V_I x')(\delta\dot{x} + V_I \delta x') - EI x'' \delta x'' + T x' \delta x' \} dz dt$$

Since

$$\delta\dot{x} = \frac{\partial}{\partial t}(\delta x), \quad \delta x' = \frac{\partial}{\partial z}(\delta x), \quad \delta x'' = \frac{\partial^2}{\partial z^2}(\delta x),$$

each term may be integrated by parts so as to eliminate the various derivatives of δx .

Performing integration by parts, we obtain

$$\int_{t_1}^{t_2} \int_z^z \{ m_t \ddot{x} + 2m_f V_1 \dot{x}' + m_f V_1^2 x'' + (EI x'')'' - (T x')' \} \delta x dz dt = 0$$

where all the integrated terms have disappeared because of the boundary conditions. Since δx is arbitrary, the only way for the integral to be zero is for the expression in bracket to be zero. Equating this to zero gives the differential equation.

$$m_t \ddot{x} + 2m_f V_1 \dot{x}' + m_f V_1^2 x'' + (EI x'')'' - (T x')' = 0$$

Combining the external hydrodynamic force and the lateral component of the difference of the hydrostatic pressure, we obtain Eq. (2.68) and (2.69).

APPENDIX B DERIVATION OF FINITE ELEMENT MODEL OF LINEAR SYSTEM

To produce the weak form, multiply the residual error function $R(x_3)$ by a sufficiently smooth test function, integrate the product by parts, and equate the result to zero. The residual error function of Eq. (3.54) is given by

$$R(x_3) = m_f \ddot{x}_1 + 2m_f V_1 \dot{x}_1' + m_f V_1^2 x_1'' + (EI x_1'')'' - w x_1' - T_e x_1'' - a_4 \rho_w D_O u_c (\dot{w} - \dot{x}_1) p(x_3) + \frac{1}{2} \rho_w u_c D_o C_D \dot{x}_1 (1 - p(x_3))$$

and test function $v(x_3) = \bar{x}_1(x_3)$.

The variational statement is given by

$$\int_0^l R \bar{x}_1 dx_3 = \int_0^l [m_f \ddot{x}_1 + 2m_f V_1 \dot{x}_1' + m_f V_1^2 x_1'' + (EI x_1'')'' - w x_1' - T_e x_1'' - a_4 \rho_w D_O u_c p(x_3) \dot{w} + \{ \frac{1}{2} \rho_w u_c D_o C_D (1 - p(x_3)) + a_4 \rho_w D_O u_c p(x_3) \} \dot{x}_1] \bar{x}_1 dx_3 = 0 \quad (B.1)$$

Integrating terms in the right side of above equation by parts and writing them,

$$\begin{aligned} \int_0^l m_f V_1^2 x_1'' \bar{x}_1 dx_3 &= m_f V_1^2 x_1' \bar{x}_1 \Big|_0^l - \int_0^l m_f V_1^2 x_1' \bar{x}_1' dx_3 \\ \int_0^l T_e x_1'' \bar{x}_1 dx_3 &= T_e x_1' \bar{x}_1 \Big|_0^l - \int_0^l T_e x_1' \bar{x}_1' dx_3 \\ \int_0^l (EI x_1'')'' \bar{x}_1 dx_3 &= (EI x_1'')' \bar{x}_1 \Big|_0^l - (EI x_1'') \bar{x}_1' \Big|_0^l + \int_0^l (EI x_1'') \bar{x}_1'' dx_3 \end{aligned} \quad (B.2)$$

Substituting (B.2) into (B.1), we obtain Eq. (3.58), i.e., the weak form of Eq. (3.54). Applying same procedure for Eq. (3.55) and (3.56), we also the weak forms of those equations as seen in Eq. (3.59) and (3.60).

Partitioning whole domain into a number of finite elements, Eq. (B.1) becomes

$$\begin{aligned} & \sum_{i=1}^N \int_{x_{3i}}^{x_{3i+}} [m_i \ddot{x}_1 \bar{x}_1 + 2m_f V_1 \dot{x}'_1 \bar{x}_1 - m_f V_1^2 x'_1 \bar{x}_1 + (E L x''_1) \bar{x}'_1 + (T_e x'_1) \bar{x}'_1 \\ & + \{a_4 \rho_w D_O u_e p(x_3) + \frac{1}{2} \rho_w u_e D_O C_D (1 - p(x_3))\} \dot{x}_1 \bar{x}_1] dx_3 + m_f V_1^2 x'_1 \bar{x}_1 \Big|_0^I \\ & + \{-(T_e x'_1) \bar{x}_1 + (E L x''_1)' \bar{x}_1 - (E L x''_1) \bar{x}'_1\} \Big|_0^I = \sum_{i=1}^N \int_{x_{3i}}^{x_{3i+}} a_4 \rho_w D_O u_e p(x_3) \dot{w} \bar{x}_1 dx_3 \end{aligned}$$

Based on Galerkin's approximation, replace above problem by a finite dimensional problem to find the approximate solutions x_{1h} , x_{2h} , and $w_h \in H^h$ such that

$$\begin{aligned} & \sum_{i=1}^N \int_{x_{3i}}^{x_{3i+}} [m_i \ddot{x}_{1h} \bar{x}_{1h} + 2m_f V_1 \dot{x}'_{1h} \bar{x}_{1h} - m_f V_1^2 x'_{1h} \bar{x}_{1h} + (E L x''_{1h}) \bar{x}'_{1h} + (T_e x'_{1h}) \bar{x}'_1 \\ & + \{a_4 \rho_w D_O u_e p(x_3) + \frac{1}{2} \rho_w u_e D_O C_D (1 - p(x_3))\} \dot{x}_{1h} \bar{x}_{1h}] dx_3 + m_f V_1^2 x'_{1h} \bar{x}_1 \Big|_0^I \\ & + \{-(T_e x'_{1h}) \bar{x}_{1h} + (E L x''_{1h})' \bar{x}_{1h} - (E L x''_{1h}) \bar{x}'_{1h}\} \Big|_0^I \\ & = \sum_{i=1}^N \int_{x_{3i}}^{x_{3i+}} a_4 \rho_w D_O u_e p(x_3) \dot{w}_h \bar{x}_{1h} dx_3 \end{aligned} \quad (B.3)$$

Approximated solutions and test functions are given from the implementation of Hermite polynomial as basis functions.

$$\begin{aligned} x_{jh}(x_3, t) &= \sum_{j=1}^4 x_{j\#}(t) N_i(x_3) \\ \bar{x}_{jh}(x_3, t) &= \sum_{j=1}^4 \bar{x}_{j\#}(t) N_i(x_3) \\ w_h(x_3, t) &= \sum_{j=1}^4 w_j(t) N_i(x_3) \quad j = 1, 2 \end{aligned} \quad (B.4)$$

The substitution of Eq. (B.4) into (B.3) introduces the series form of the weak form. Factorizing the coefficient of test function from the series form, we have the compact form such as Eq. (3.44). And then, we can obtain the matrix dynamic equilibrium equation (3.61) through the appropriate choice of the coefficients of test function as shown in

section 3.4. Applying same procedures for Eq. (3.59) and (3.60) as introduced above, we can also derive the corresponding matrix equation (3.62) and (3.63).

BIBLIOGRAPHY

- Ashley, H. and Haviland, G., "Bending Vibrations of a Pipe Line Containing Flowing Fluid," *Journal of Applied Mechanics*, Vol.17, 1950, pp.229-232.
- Bathe, K.J. and Wilson, E.L., *Numerical Methods in Finite Element Analysis*, Prentice Hall, Inc., Englewood Cliffs, New Jersey, Part III, 1976.
- Benjamin, T.B., "Dynamics of System of Articulated Pipes Conveying Fluids, Part 1 and 2," *Proceedings of the Royal Society of London, Series A*, 261, 1961, pp.457-499
- Bernitsas, M.M. and Kokarakis, J.E., "Large Deformation Three-Dimensional Static Analysis of Deep Water Marine Risers," *Applied Ocean Research*, Vol.7, No.4, 1985, pp.178-187.
- Blevins, R.D., *Flow Induced Vibration*, Van Nostrand Reinhold Co., New York, 1990.
- Bruce, E.B. and Michael, F.M., "Nonlinear Dynamic Analysis of Coupled Axial and Lateral Motion of Marine Risers," *Offshore Technology Conference*, OTC 2776, Houston, Texas, 1977, pp.403-412.
- Burke, B.G., "An Analysis of Marine Risers For Deep Water," *Journal of Petroleum Technology*, April 1974, pp.455-465.
- Chakrabati, S.K. and Frampton, R.E., "Review of Riser Analysis Techniques," *Applied Ocean Research*, Vol.4, No.2, 1982, pp.73-90.
- Chen, B.C.M., "A Marine Riser with Internal Flow-Induced Vibration," *Offshore Technology Conference*, OTC 6893, Houston, Texas, 1992, pp.183-190.
- Chen, S.S., "Out of Plane Vibration Stability of Curved Tubes Conveying Fluid," *Journal of Applied Mechanics*, Vol.40, 1973, pp.362-368.
- Chung, J.S., "Hydrodynamic Forces on a Marine Riser: A Velocity Potential Method," ASME Paper No. 76-Pet-81 at the Petroleum Mechanical Engineering Conference, Mexico City, Mexico, Sept. 1976.

- Chung, J.S., "Nonlinear Transient Motion of Deep Ocean Mining Pipe," ASME Journal of Energy Resources Technology, Vol.103, Mar.1981, pp.2-10.
- Chung, J.S. and Fellipa, C.A., "Nonlinear Static Analysis of Deep Ocean Mining Pipe - Part II: Numerical Studies," ASME Journal of Energy Resources Technology, Vol.103, Mar.1981, pp.16-25.
- Dareing, D.W., and Huang, T., "Natural Frequencies of Marine Drilling Risers," Journal of Petroleum Technology, July 1976, pp.813-818.
- Dareing, D.W. and Huang, T., "Marine Riser Vibration Response Determined by Modal Analysis," ASME Journal of Energy Resources Technology, Vol.101, Sep.1979, pp.159-166.
- Duncan, W.J., "Reciprotation of Triply Partitioned Matrices," J. Roy. Aeron. Soc., 60: 1956, pp. 131-132.
- Fellipa, C.A. and Chung, J.S., "Nonlinear Static Analysis of Deep Ocean Mining Pipe - Part I: Modeling and Formulation," ASME Journal of Energy Resources Technology, Vol.103, Mar.1981, pp.11-15.
- Gardner, T.N. and Kotch, M.A., "Dynamic Analysis of Risers and Caissons by the Element Method," Offshore Technology Conference, OTC 2651, Houston, Texas, 1976, pp.405-421.
- Garrett, D.L., "Dynamic Analysis of Slender Rods, " ASME Journal of Energy Resources Technology, Vol.104, Dec.1982, pp.302-306.
- Gosse, C.G. and Barksdale, G.L., "The Marine Riser - A Procedure for Analysis," Offshore Technology Conference, OTC 1080, Houston, Texas, 1969, pp.109-116.
- Hibbler, R.C., Engineering Mechanics, Dynamics, Fourth Edition, Macmillan, New York, 1986.
- Hill, J.L. and Davis C.G., "The Effect of Initial Forces on the Hydroelastic Vibration and Stability of Planar Curved Tubes," Journal of Applied Mechanics, Vol.41, 1974, pp.355-359.
- Hill, M.N., The Sea, Vol.1, Interscience Publishers, New York, 1962.
- Homes, P.J., "Pipes Supported at Both Ends Cannot Flutter," Journal of Applied Mechanics, Vol 45, 1978, pp. 619-622.
- Housner, G.W., "Bending Vibrations of a Pipe Line Containing Flowing Fluid," Journal of Applied Mechanics, Vol.19, 1952, pp.205-208.

- Huang, T. and Chucheepsakul, S., "Large Displacement Analysis of a Marine Riser," ASME Journal of Energy Resources Technology, Vol.107, Mar.1985, pp.54-59.
- Iwan, W.D., and Blevins, R.D., "A Model for Vortex-Induced Oscillations of Structure," ASME Journal of Applied Mechanics, Vol. 96, 1974, pp.581-586.
- Kim, Y.C., "Natural Frequencies and Critical Buckling Loads of Marine Riser," ASME Journal of Offshore Mechanics and Arctic Engineering, Vol.110, Feb. 1988, pp.2-8.
- Kim, Y.C. and Triantafyllou, M.S., "The Nonlinear Dynamics of Long, Slender Cylinders," ASME Journal of Energy Resources Technology, Vol.106, June 1984, pp.250-256.
- Kokarakis, J.E. and Bernitsas, M.M., "Nonlinear Three Dimensional Dynamic Analysis of Marine Risers," ASME Journal of Energy Resources Technology, Vol.109, Sep.1987, pp.105-111.
- Konuk, I., "Application of an Adaptive Numerical Technique 3-D Pipeline Problems with Strong Nonlinearities," ASME Journal of Energy Resources Technology, Vol.104, Mar.1982, pp.58-62.
- Labbe, J.R and Nikkel, K.G., "Sensitivity of Marine Response to the Choice of Hydrodynamic Coefficients," Offshore Technology Conference, OTC 4592, Houston, Texas, 1983, pp.103-112.
- Leira, B.J. and Remseth, S.N., "A Comparison of Linear and Nonlinear Methods for Dynamic Analysis of Marine Risers," Proceedings of the 4th International Conference on Behaviour of Offshore Structures, 1985, pp.383-394.
- Love, A.E.H., A Treatise on the Mathematical Theory of Elasticity, 4th ed., Dover Publications, New York, 1927.
- McNamara, J.F. and Lane, M., "Practical Modeling for Articulated Risers and Loading Columns," ASME Journal of Energy Resources Technology, Vol.106, Dec.1984, pp.444-450.
- Moe, G. and Chucheepsakul, S., "The Effect of Internal Flow on Marine Risers," 7 th International Conference on Offshore Mechanics and Arctic Engineering, Tokyo, Japan, 1988, pp.375-382.
- Nordgren, R.P., "On Computation of the Motion of Elastic Rods," ASME Journal of Applied Mechanics, Vol. 96, Sep.1974, pp.777-780.
- Nordgren, R.P., "Dynamic Analysis of Marine Risers with Vortex Excitation," Journal of Energy Resources Technology, Vol. 104, Mar. 1982, pp.14-19.

- O'Brien, P.J. and McNamara, J.F., "Three Dimensional Nonlinear Motions of Risers and Offshore Loading Towers," ASME Journal of Offshore Mechanics and Arctic Engineering, Vol.110, 1988, pp.232-237.
- Oden, J.T., Finite Elements of Nonlinear Continua, McGraw Hill Co., New York, 1972.
- Paidoussis, M.P., "Dynamics of Flexible Cylinders in Axial Flow, Parts 1 and 2," Journal of Fluid Mechanics, Vol. 26, 1966, pp. 717-751.
- Paidoussis, M.P., "Dynamics of Tubular Cantilevers Conveying Fluid," Journal of Mechanical Engineering Science, Vol. 2, 1970, pp. 85-103.
- Paidoussis, M.P., "Dynamics of Finite-Length Tubular Beams Conveying Fluid," Journal of Sound and Vibration, Vol.106, 1986, pp.311-331
- Paidoussis, M.P., and Issid, N.T., "Dynamic Stability of Pipes Conveying Fluid," Journal of Sound and Vibration, Vol. 33, 1974, pp. 267-294.
- Paidoussis, M.P., and Moon, F.C., "Nonlinear Fluidelastic Vibrations of a Flexible Pipe Conveying Fluid," Journal of Fluids and Structures, Vol. 2, 1987, pp.567-591.
- Robert, D. Cook, Concept and Applications of Finite Element Analysis, John Wiley & Sons, New York, 1981.
- Safai, V.M., "Nonlinear Dynamic Analysis of Deep Water Risers," Applied Ocean Research, Vol.5, No.4, 1983, pp.215-225.
- Sexton, R.M. and Agbezuge, L.K., "Random Wave and Vessel Motion Effects on Drilling Riser Dynamics," Offshore Technology Conference, OTC 2650, Houston, Texas, 1976, pp.391-404.
- Spanos, P.T.D. and Chen, T.W., "Vibrations of Marine Riser Systems," Journal of Energy Resources Technology, Transactions of the ASME, Vol. 102, Dec. 1980, pp.203-213
- Wu, M.C. and Lou, J.Y.K., "Effects of Rigidity and Internal Flow on Marine Riser Dynamics," Applied Ocean Research, Vol.13, No.5, 1991, pp.235-244.

BIOGRAPHICAL SKETCH

The author was born on August 17, 1959, in Seoul, Korea, where he has received all his education, beginning with primary school almost 30 years ago and concluding with his master degree from the Engineering School of Yonsei University. As the eldest son in a family where sons are rare, he has always had a great deal of family responsibility. Because of that, his father encouraged pursuing a high level of education instead of high risk sports such as rock climbing or cycling.

After graduation of high school, he entered in the Department of Civil Engineering at Yonsei University. During his college life, he decided to continue his study and enrolled in the Graduate School of the same university. There, he received a Master of Structural Engineering in 1985. After serving military duty for his country, he came to the United States in order to get a higher degree.

He enrolled at the University of Florida to pursue a doctoral degree in coastal and oceanographic engineering. This time, a lovely wife and a pretty daughter accompanied him. Wherever he goes, another challenge is waiting for him and he will overcome it no matter how hard it is. He will always struggle to be an excellent researcher.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Robert G. Dean, Chairman
Graduate Research Professor
of Coastal and Oceanographic Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Hsiang Wang
Professor of Coastal and Oceanographic
Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



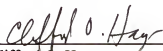
D. Max Sheppard
Professor of Coastal and Oceanographic
Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Robert J. Thieke
Assistant Professor
of Coastal and Oceanographic Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.


Clifford O. Hays
Professor of Civil Engineering

This dissertation was submitted to the Graduate Faculty of the College of Engineering and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August 1994


for Winfred M. Phillips
Dean, College of Engineering

Karen A. Holbrook
Dean, Graduate School